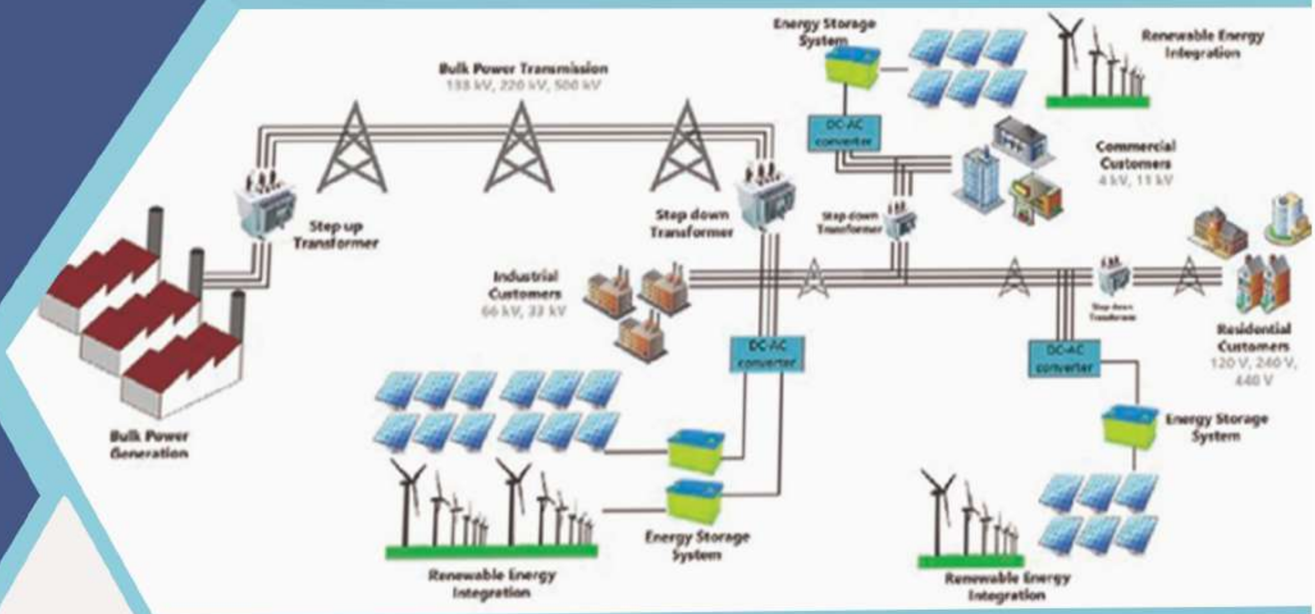
Power Market with Congestion Management



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PREFACE

"Power Market and Congestion Management" delves into the intricate dynamics of electricity markets and the challenges posed by congestion in transmission networks. It explores the foundational principles of power trading, market structures, and pricing mechanisms, offering insights into how energy flows are optimized. The book emphasizes the critical role of congestion management techniques, such as locational marginal pricing (LMP), demand-side solutions, and grid enhancements, in ensuring efficient and reliable power delivery. It examines regulatory frameworks, economic implications, and technological advancements shaping modern electricity markets. By addressing issues like renewable integration, grid constraints, and market competition, this work provides a comprehensive guide for policymakers, engineers, and market participants. With real-world case studies and practical strategies, it equips readers to navigate the complexities of power systems while fostering sustainable energy practices. A must-read for anyone seeking a deeper understanding of the evolving energy landscape.

CHAPTER ONE

Introduction

Electrical power is probably one of the most important elements of modern society. Since the discovery of electricity and the invention of electric dynamo and incandescent light bulb, people's life style has been changed significantly. Electric power systems provide a clean and convenient energy to light houses and drive machine motors. Electric energy has become a necessity of daily life.

Pearl Street Station, the first power station established in 1882 by Thomas Edison, supplied DC power for lighting streets within a short distance from the station. Due to the excessive power losses and voltage drop, the low voltage of DC system became the bottleneck of long-distance electricity delivering. The invention of AC transformer made it possible to raise the voltage to a higher level for long distance power transmission. The advantages of AC power systems were obvious, and AC power systems were adopted all over the world. In recent decades, HVDC/EHVDC has developed very fast, which helps to transfer electric power in a more efficient way through long distances. In the past century, electric power system had a significant development. Large-scale centralized power plants were established due to the economies of scale. High-voltage transmission and distribution lines were installed to deliver electric energy to every corner of the world. Power lines of different voltage levels form the power grid, which interconnects power plants and electricity users in different areas. The interconnection of power grids makes the overall power generation and transmission more economical and reliable. Through interconnection, less expensive generators can generate more power to supply customers at expensive areas, and fewer generators are engaged as reserves for peak load periods in an interconnected system.

The power system operator monitors and controls the system and maintains the system to be operated in a safe and reliable way. In addition to system reliable operations, the system operator is responsible for power system *economic operation*. The purpose of economic operation is to dispatch electric power generators with the minimum generation cost while satisfying the electricity demand. As the amount of fuel consumptions for a power system is huge, the savings on fuels in a small percentage by economic operation could result in a large amount of savings in generation costs.

Electric power is generated by different types of generators. According to fuel types, conventional power plants could be categorized as coal-fired power plant, oil-fired power plant, gas turbines, nuclear power plant, and hydro power station. With today's emphasis on energy and environmental considerations, renewable energy, such as wind power-, solar-, and geothermal-based generations, is increasing significantly. For different types of generators, generation costs are different due to fuel prices and generation technologies. Every generating unit has its unique generation cost characteristics. In a power system, the total available generation capacity is bigger than the total demand for almost anytime. To supply a given amount of electricity demand, there are more than one options/combinations of generating units for the system operator to dispatch the generators. The total generation cost differs for different combinations of generating units and outputs. It would be more economical to find the option/combination with lower cost to supply the electricity demand. This is the basic principle of power system economic operation. Power network constraints also need to be considered for economic operation.

Conventional power system economic dispatch is implemented on the basis that all generators in the network are owned by one power company, and the system operator knows the cost curves of all generators. The system operator searches for a least-cost solution for dispatching generating units in the system.

Power industry has been changed since power deregulation in 1990s. Generations are separated from the transmission network. Most generators are independent power providers owned by generation companies. Independent system operators (ISOs) or transmission system operators (TSOs) are responsible for system reliable and economic operations. The generation cost of each generator is not known to the ISO/TSO. Electric energy is traded in the electricity market. In the market, the concept of economic dispatch is extended with market operation. The market design, clearing procedure, and settlement process affect the way electricity is traded and the generation dispatched. Besides, power network constraints need to be considered in the market operation.

The purpose of the book is to provide a systematic understanding of power system economic operations in traditionally vertically integrated systems and market operations in deregulated power system. The principles of economic dispatch, unit commitment, and optimal power flow and their mathematical models will be introduced. Then, the market mechanisms and mathematical models for energy market and ancillary services will be presented. In the end, electricity financial market and low carbon electricity market operation will be introduced.

The book is targeted to senior students and postgraduate students in electric power engineering and energy engineering, researchers and engineers in the area of power system economic operations and electricity markets, system operators, electricity marketers, electricity retailers, and other electricity market participants.

CHAPTER TWO

Economic operation in power systems

2.1 Introduction of power system operation

The electric power system is one of the most complicated systems in the world, due to its large size, real-time operation, and the large number of customers involved. Conventionally, a power system can be subdivided into four parts: generation, transmission, distribution, and customer services. A vertically integrated power company owns generation, transmission, and distribution facilities and provides customer services. Electricity generated by generators flows through transmission system and distribution network to customers.

Electric power is generated by different types of generators. Most of them are centralized large-scale power plants located far from customers. To transmit electricity over a long distance, high voltage is needed. Apparent power transmitted is the product of voltage and current. Transmitting the same amount of power, using a higher voltage level results in a lower current in the power line, and hence a lower voltage drop (I*Z) and lower losses (I^2*R). Thus, transmission lines with higher voltages can transmit more power. The terminal voltages of generating units are mostly not high enough for long distance power delivery. Voltage is stepped up by transformers in power stations. Power is transmitted through high-voltage/extra high voltage transmission networks to load centers. Then, voltage is stepped down through distribution substations to low voltage levels for power distributions. Distribution substations are usually close to end users. Distribution lines/power feeders connect residential and commercial customers with distribution substations. Power grid connects millions of electricity end users and industry users to generators located at different places. Geographically, a power grid covers a very wide area. Electricity flowing in the power grid follows physical laws. The system operator is responsible for monitoring and controlling the system to ensure that it is operated in a safe and reliable way all the time. The most important task for the operator is to maintain the real-time power balancing of the power grid. For example, when any customer switches on his/her electric device, electricity flows instantaneously to the device and powers it up. Power generation is adjusted simultaneously to follow the load change. This is the real-time power balancing between electric power generations and electricity demands.

In the traditional power system, most customer behavior is invisible and uncontrollable to the system operator. Although some customers could respond to the system operator with recent developments of smart grid technologies, such as distributed generation, communication and demand response, and so on, the response of demand to the operator is still limited. To the system operator, most customers are uncontrollable passive customers, whereas outputs of most conventional generators are controllable. Therefore, the system operator controls and adjusts power outputs of generators to follow load changes to maintain real-time power balancing. The traditional operation paradigm is *generation following load*. Millions of small diverse customers' electricity consumptions are balanced by controlling the outputs of several large generators in the system.

Changes in loads are the major uncertainties to the conventional power system operation. Later, with the installations of more renewable energy generations, such as wind power and photovoltaic panels, the intermittencies of renewable energy become part of the uncertainties to power system operations. For the system operator, an accurate forecast of load is quite important. According to time scales, load changes could be categorized as (1) long-term load changes caused by load growth; (2) mid-term load changes due to seasons, weathers, or other reasons; (3) short-term load variations of consumption patterns; and (4) very short-term load fluctuations. The trend of load changes in a long term could be forecasted based on economic growth. In different seasons and weather conditions, electricity consumptions are different, it is, however, forecastable using historical data and weather forecasting results as the basis. Short-term load variations and load fluctuations are hard to accurately forecast.

The power system operation has similar time periods for system planning and operation, which matches the time of load changes. Transmission line constructions and generating unit installations are planned years in advance according to forecasted load growth and system reliability requirements while considering transmission capabilities. Generation schedules for all generating units are made in advance. In some systems, the amount of energy that will be produced by a generating unit is scheduled 1 year or months in advance. When time is closer, detailed generation scheduling is made by considering the load forecasting results and network constraints. Generating units are committed days or weeks ahead considering unit maintenance schedules, minimum ON/OFF requirements, unit startup costs, system reserve requirements, and so on. Once units are committed, day-ahead and hourly-ahead generation schedules are procured by the system operator using more accurate load forecasting results, while considering network constraints, security constraints, generation costs, and other factors. However, even the hourly ahead scheduling is not accurate enough to match the real-time load, as load fluctuates and could be different from forecasted values. Due to real-time load changes, outputs of generating units in the system need to be adjusted in real time to respond to fast load variations.

In a power system, the system frequency is an indicator of the balance of generation and load. When the total generation and system load is balanced, the frequency of the moment is equal to the nominal frequency. When generation exceeds the load, the frequency at the instant is higher than the nominal frequency. Also, when generation is not sufficient to supply load, the frequency at the instance is lower than the nominal frequency. The target of the system operator is to control and maintain the frequency at the nominal value, 50 or 60 Hz, which is the index of real-time power balancing. *Frequency control* is the approach used in real-time operation for power balancing. Most generating units are equipped with the function *automatic generation control* to react to real-time frequency signals and adjust their generation outputs automatically. In addition to the generation scheduling and frequency control, the system operator needs to consider many other issues, such as power system stability, reactive power and voltage control, blackout restoration, and so on.

Whether in long-term generation planning, or in mid-term or short-term generation scheduling, generation costs are the major concern in addition to power balancing. A more economical combination of generating units and a less expensive generation schedule can save a large amount of fuels and generation expenses. This book focuses on the methods of operating the power system in a more economical way, in other words, *power system economic operation*. The mathematical methods for obtaining most economical/optimized generation schedules in a power system will be introduced. How to operate the power system and make generation schedules in an electricity market will be discussed. Electricity pricing, electricity market models, and market settlements will also be presented in the book.

2.2 Development of economic operation

The system operator, at the early stage of power system development, has already realized that once a system has more than one generating units, the fuel costs would be different for different generation schedules made for the units. This is mainly because that each generating unit has its unique fuel consumption characteristics, which is a nonlinear function of power output. The fuel consumption of a generating unit depends on its fuel consumption curve and the operating point. For a given electricity load, fuels consumed by different generators are different. If one generator is enough to supply the load, the generator with lowest fuel consumption will be selected. If multiple generators are selected, there will be multiple options to allocate generations to the selected generators. Different generation allocation options result in different results of fuel consumptions/costs. If the option with the least fuel consumption/cost is fortunately selected, the generation schedule is an economical schedule. Of course, the system operator can enumerate all options and find out which option is the most economical. However, the enumeration method may not be practical for a system with a big number of generators. The number of generation allocation options could be too large to be enumerated in a reasonable time. To obtain the most economic generation schedule, optimization methods can be used to minimize the total fuel consumption/cost while considering system operation requirements. This is the purpose of power system economic operation.

Economic generation schedules are usually made days or hours before the real-time energy delivery. Use 1 day as an example, electric demand (in MW) is first forecasted for the 24 hours of the day. For each hour, the forecasted load is assumed as given. The system operator needs to decide how much electricity should be generated by each generator in each hour, so that the total generation in the hour equals to the forecasted load plus losses. It would be better if the total cost is the most economical. Mathematically, this is an optimization problem. Due to the big size of power system and its nonlinear characteristics, the problem comes out to be a complicated optimization problem. Before computer was developed, it was difficult to solve the complicated nonlinear optimization problem by hand. In 1930s, electrical engineers figured out the basic principles of *economic dispatch* (ED), which has been used to obtain economic generation schedules. The method is still in use currently in some locations.

The key of economic dispatch is *equal incremental cost function*. It was obtained by electrical engineers in 1930s after exploring the economical allocation of generations to generators. People noticed that, similar to other commodity goods, the value/price of electricity generation depends on its incremental cost, which is the additional cost of producing one more unit of electricity from its current output level. This is also called *marginal cost*. The significance of economic dispatch is that engineers have proved that the total cost is the minimum if all generators in the system are operated at the output points with the same incremental cost. Economic dispatch results obtained with the equal incremental cost principle satisfy the constraint that the total generation equals to the total load during the hour.

Transmission losses cannot be avoided in any power system. When equal incremental cost function was derived, only generation characteristics of generators were studied and transmission lines were not considered. Later in 1940s, electrical engineers tried to use a quadratic function to simplify and represent transmission losses occurring in the system. The classical economic dispatch method was amended by including transmission losses. The equal incremental cost was found to be subject to the constraints that the total generation equals the total load plus the simplified loss. The new equal incremental cost function with *penalty factor* that represents the impacts of losses was obtained, and it is called *coordination equation.*

Equal incremental cost function and coordination equation have been well applied for economic operation ever since they were developed. The economic dispatch results obtained by them are economically effective and the calculation procedure is straightforward. The only disadvantage is that transmission losses were simplified due to the lack of accurate model for transmission network.

In 1950s to 1960s, with the increase in voltage levels and the interconnection of power grids, power systems became much more complicated. It was necessary to find an accurate way to calculate the active power and reactive power that flow in the lines. As AC power system is a nonlinear system, the simple Ohm's Law is not suitable. Using Kirchhoff's Law, electrical engineers wrote active power and reactive power balancing equations for all nodes of the system. They are power balancing equations, also called *power flow equations*. The equations are able to mathematically represent the power network. This is one of the big steps in the development of power system analysis. Power flow equations are nonlinear equations. Newton Raphson method is one of the effective ways to solve the equations. Starting with initial values, Newton Raphson method provides searching directions for each iteration to reach a converged solution. The number of iterations could be big if the system is large and the initial values are far from the converged solution. The development of computer technologies in 1960s made it possible to solve the power flow equations. Then, power network could be represented mathematically, and power flow was calculated by computer programming. This led to a new era for power system analysis.

In the area of power system economic operation, J. Carpentier must be mentioned due to his contributions in the development of optimal power flow (OPF). On the basis of the traditional economic dispatch approach, Carpentier replaced the simplified power balancing equation with nonlinear power flow equations to represent the full power network in the optimization problem. Optimal power flow is an optimization problem that minimizes the total cost or the fuel consumption subject to full network constraints. OPF is a nonlinear programming (NLP) problem. The solution of optimal power flow is better than economic dispatch due to its consideration of accurate full network model and losses.

Optimal power flow is a nonlinear programming problem owing to the constraints of full power network, which is an extremely nonlinear system. It had been a challenge to solve the OPF problem at the beginning when it was first formulated. The problem was identified as a nonconvex one. During old days, there were difficulties in modelling discrete variables and finding local minima or stable converged solutions. Many linearization algorithms had been tried to solve the problem. Since 1990s, the fast development of computer technologies and optimization algorithms has changed the situation. More powerful optimization algorithms and computing algorithms are applied to solve the defined OPF problem with fast computers. Solution convergence and modeling of discrete variables are no longer that difficult.

Powerful computers with supercomputing technologies provide the possibilities of solving the complicated nonlinear programming problem in a short time. Now, in the power control center, optimal power flow problem need to be solved very frequently every day. It has become a very useful tool in power system planning, generation scheduling, and electricity market clearing. With mixed-integer programming technology, optimal power flow is applied to unit commitment problem, which involves the switching on or switching off of generating units during multiple time periods.

OPF model has been extended to include various constraints. The OPF model considering transmission thermal limits, transmission stability, and voltage constraints and contingencies are referred as security-constrained OPF (SCOPF). The objective function of OPF can be modified to form different optimization problems, for example, reactive power optimization, electricity market clearing, social welfare, and so on.

Traditional power system economic operation is run by the system operator of vertically integrated power company. The system operator knows generation cost functions of their generating units. As electricity retail prices to customers are regulated, the objective function of optimal power flow for the traditional system economic operation is to minimize the total generation cost of all generating units.

Power industry deregulation and electricity market that started in 1990s have changed the operation paradigm. Economic operation changed from a centralized dispatch model to a market-based model. Generation costs are not known to the system operator any more. Electricity retail prices may not be regulated as before. Generator offer prices and customer bid prices are available to the system operator to decide generation schedules and clear the market. Modified OPF minimizing the total payment is used by the system operator to determine uniform market prices or nodal prices for an electricity market. The full network constraints and security constraints of OPF provide a perfect way to combine power grid real-time security operation requirements with market-clearing mechanisms.

The power industry has been a monopoly industry since it was first developed in late nineteenth century. The deregulation started in 1990s introduced competition to generators.

The centralized power dispatch problem in classical economic operation has been changed to a decentralized market mechanism that includes long-term contracts, electric energy market, and ancillary service market. Independent power producers and large customers are the major participants of the market. In the coming years, with the development of smart meter technologies and demand response programs, electricity demand and consumption patterns of an individual small customer could also be considered and optimized in the optimization problem of electricity market. Market prices will provide incentives for customers to respond to the system operator and participate in demand response programs and electricity markets. In the future, both generators and customers will be involved in power system economic operation. Not only generations of generators but also electric demand of customers will be scheduled for system operation and optimized in the electricity market.

2.3 Incentives of economic operation

The effect of economic operation is extremely huge. For example, for a system with a peak load of 92 GW, and the annual total generation of 512,000 GWh, if the fuel cost for power generation is \$30 per MWh, the annual total generation cost is around 15.6 billion dollars. Even if the economic operation algorithm can help reduce the generation cost by only 0.5%, the saving is still huge. It could be around 78 million dollars. The economic effect is significant if economic operation is applied to all power systems.

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CHAPTER THREE

Power generation costs

The cost of electric power generation is the basis for power system economic dispatch and market analysis. In this chapter, we will introduce the generation costs for different types of generators.

3.1 Load cycles

Electricity is generated and delivered to customers in real time. The amount of electricity consumption by customers is called *electricity demand* or *electricity load*. For a very long time, most customers paid electricity bills with fixed tariffs. There is almost no communication between the utility and customers except for monthly electricity bill payments. A customer turns on switches, his/her devices or lights will be powered on instantaneously. For small customers, no one is required to inform the utility of his/her electricity usages in advance. In fact, it is the utility's responsibility to forecast the electricity demand in advance, and adjust the outputs of generating units to follow the load changes in real time. One of the major tasks of the power system operator is to maintain the real-time balancing between system generations and the loads. It is important for the system operator to estimate the amount of electricity that will be used by customers beforehand. So, load forecast is necessary for the system operator.

The electricity load in a power system has its own profile and characteristics. As human being's activities follow the periodical changes (years, seasons, months, and days) of the Nature, electricity consumptions follow certain periodical cycles. Use one day as an example, people wake up in the morning and

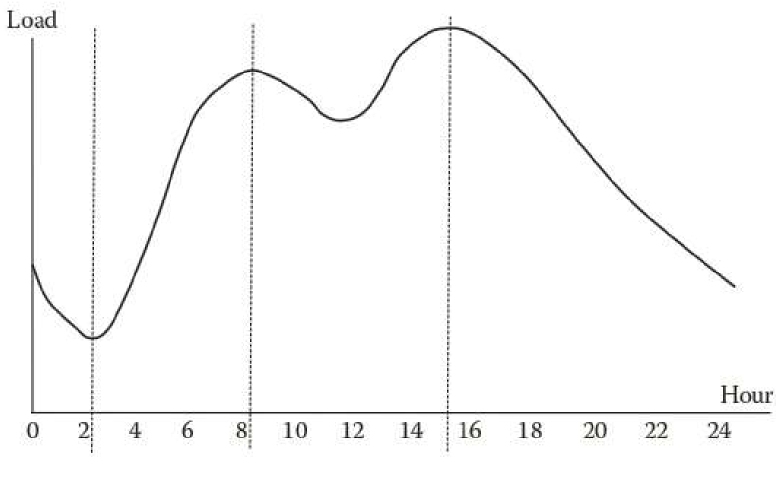

FIGURE 3.1 Daily load curve for residential and commercial customers.

switch on various electrical devices, so the demand of electricity in the morning is high. Another time period with high demand is from afternoon to evening, when all lights are on and people cook and watch TVs at home. After midnight, most people are sleeping, the electricity demand is low until next morning. The electricity consumption pattern repeats every day for residential customers and commercial customers. The daily load curve for residential and commercial customers is shown in Figure 3.1. Moreover, people's activities during weekdays are different from those at weekends, so the load pattern during weekends is different from that during weekdays. A weekly load curve for residential and commercial customers is shown in Figure 3.2. Besides human's activities, weather changes are the other major reasons affecting electricity demand. In cold regions, electricity demand is high in the winter for heating. While in warm regions, more electricity is used for cooling in the summer.

Besides periodically changing electricity demands caused by weather changes and people's activity cycles, some electricity consumptions such as industrial loads, do not change much within 1 day nor even within the whole year. Some manufacturing factories need to keep their machines continuously running 24 hours per day and 365 days per year. Ventilation systems of commercial buildings and customer refrigerators are always on. The type of load that is switched on in most of the time is called *baseload*.

The electricity load curve (Figures 3.1 and 3.2) looks like hills. The time periods with high electricity demands are called

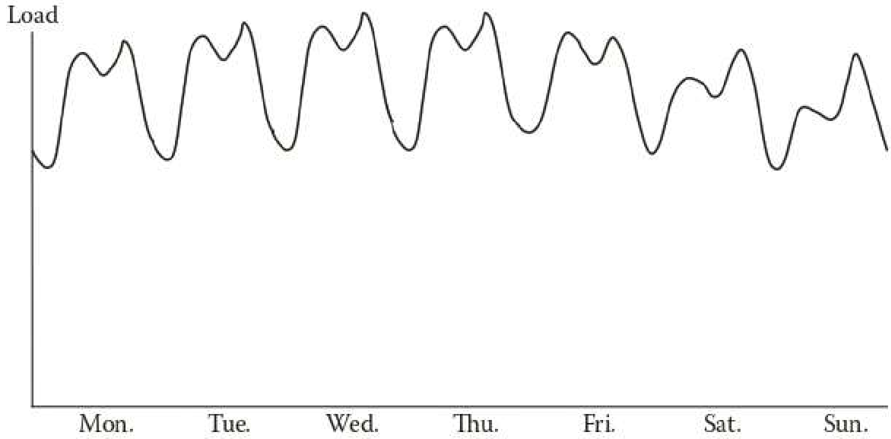

FIGURE 3.2 Weekly load curve for residential and commercial customers.

peak hours, while the time periods with low demands are called valley (or off-peak) hours. Peak periods include morning peak and afternoon peak. The electricity loads during peak hours are called peak loads, while loads during off-peak hours are off-peak loads. In real-time operation, the electricity load may change in a short term from off-peak load level to peak load level, for example, electricity demand increases in the morning from 8 a.m. to 9 a.m.

In a long term, electricity demand usually grows gradually. Economic growth of the area normally affects the growth rate of electricity demand. Utilities forecast long-term load increases for long-term system planning. Enough generating capacities need to be invested and installed to cover the highest peak demand of a year plus the required generation reserve margin. The generation reserve is required for the purpose of system reliability operation. On the other hand, new transmission capacities are planned on the basis of long-term load forecasts and the results of generation planning.

Besides long-term load forecast, short-term and ultrashort-term load forecasts are necessary for real-time operation. Short-term load forecast estimates the load profile for future 24 hours or 7 days, and the forecast time interval is 15 minutes or 1 hour. The results of short-term load forecasts are used for day-ahead generation scheduling. Ultrashort-term forecast estimates the load in a time interval of 1 minute for future 5–30 minutes. The results are used for real-time power system control, online load flow, and contingency analysis.

Load curves for future time periods are available once load forecasts are given. Long-term power system planning, generation scheduling, and real-time system operation are implemented based on load forecast results of various time intervals. No matter it is for power system planning or generation scheduling, generation costs of generating units are the major concern for the system operator. Generation costs vary for different types of generating units. Due to load characteristics, some units are dispatched continuously to supply baseload and some fast-response units are turned on only during peak hours. Baseload units and peaking units have different generation costs besides different responding times. Both costs and technical requirements of generating units need to be considered for power system long-term planning and short-term operation.

3.2 Costs for power generations

Supply of electricity is a business that is complex and expensive. Steam turbines were commonly used in the early power stations. The technologies had been well-developed and applied by most power companies. The fuels for steam turbine-based power plants were usually coal, and sometimes oil or gas. Later, hydro power and nuclear power were developed for power generations. They are clean and their fuel costs are less. The fast growth of nuclear power has slowed down in Western countries since 1980s due to safety reasons. Then, gas turbines became popular for power stations. Combined cycle power plants with higher efficiencies use both gas turbines and steam turbines. Alternative energy technologies were developed in the end of 1970s. It was somehow because of high oil prices. The growth of alternative energy was speeded up in late 1990s for environmental concerns and global warming. Various types of renewable energy sources, such as wind power and solar energy, contribute to the electricity generation mix. In the twenty-first century, renewable energy will be the future of electric power generation.

3.2.1 Coal-fired steam power plants

The basic steam cycle power plant can use any source of heat to heat up the water in the boiler and produce steam to drive the steam turbine. The heat could come from combustions of fossil fuels, nuclear reactions, or concentrated sunlight. The high-pressure steam expands and spins the turbine, which is coupled with the generator by the shaft. The generator converts rotational kinetic energy into electrical energy. The exhausted steam is cooled down in the condenser, and then pumped back to the boiler to be reheated.

Coal-fired steam power plants provide more than half of the electric energy in the world. Centralized coal-fired power plants used to be dominant in traditional power systems due to economies of scale. The cost of electricity generation depends on capital cost of building the plant and the fuel cost. The capital cost for a coal-fired power plant is usually expensive compared to power plants burning other types of fossil fuels. The investment cost of a new coal-fired power plant with pollution-control systems is even higher. However, even taking into account the investment in pollution-control technologies, coal-fired power plants are competitive for their low fuel costs in most cases where local coal mines are available. Coal is expensive to transport. The method of coal transportation can affect the cost of coal-fired power generation.

The efficiency of a thermal power plant is usually evaluated using *heat rate*. It is the amount of thermal input (in Btu or kJ) required to generate 1 kWh of electricity. The conversion between two units, British thermal unit (Btu) and kilojoule (kJ) is 1 Btu = 1.055 kJ. The heat rate of thermal power plant is usually expressed in Btu/kWh or kJ/kWh. A smaller heat rate means less fuel is needed to generate 1 kWh of electrical output; hence, the thermal efficiency is higher. In SI unit, 1 J is the work required to produce 1 W of power for 1 second: $1\text{ kJ} = 1\text{ kW}\cdot\text{s}$, or $3{,}600\text{ kJ} = 1\text{ kWh}$. Generation efficiency η is expressed as the ratio of output electrical energy and input thermal energy.

$$\eta = \frac{\text{Output (kWh)}}{\text{Input (kWh)}} = \frac{\text{Output (kWh)}}{\left(\text{Input (kJ)}/3{,}600\text{ (kJ/kWh)}\right)}$$

$$= \frac{3{,}600\text{ (kJ/kWh)}}{\left(\text{Input (kJ)}/\text{Output (kWh)}\right)} = \frac{3{,}600\text{ (kJ/kWh)}}{\text{Heat rate (kJ/kWh)}}$$

Or

$$\eta = \frac{3{,}412\text{ (Btu/kWh)}}{\text{Heat rate (Btu/kWh)}} \tag{3.1}$$

Heat rate can be expressed using efficiency η.

$$\text{Heat rate (kJ/kWh)} = \frac{3{,}600\text{ (kJ/kWh)}}{\eta}$$

or

$$\text{Heat rate (Btu/kWh)} = \frac{3{,}412\ \text{(Btu/kWh)}}{\eta} \tag{3.2}$$

For example, if the heat rate of a power plant is 10,000 kJ/kWh (or 9,479 Btu/kWh), the efficiency of the plant is $\eta = \left(3{,}600\ \text{(kJ/kWh)}/\text{Heat rate (kJ/kWh)}\right) = (3{,}600\ \text{kJ/kWh}/10{,}000\ \text{kJ/kWh}) = 36\%$. Assume the power plant burns bituminous coal with a heating value of 12,130 Btu/lb. Consider 1 Btu/lb = 2.32 kJ/kg, the heating value is 28,262 kJ/kg. The coal rate for energy generation is coal rate = (10,000 kJ/kWh/28,262 kJ/kg) = 0.354 kg coal per kWh,which means 354 g coal is needed to generate 1 kWh electricity.

In the world, coals have different classifications and heating values due to their ingredients. Standard *coal equivalent* is used as the reference unit to evaluate calorific values of fuels: 1 kg coal equivalent corresponds to 7,000 kcal (29,308 kJ or 8.14 kWh). In the previous example, 1 kg bituminous coal (heating value 28,262 kJ/kg) corresponds to 0.964 kg coal equivalent, which is calculated by (28,262 kJ/29,308 kJ/kg) = 0.964 kg.

In a vertically integrated power system, generators, transmission, and distribution facilities are all owned by one power company and operated by the system operator. Besides capital costs, daily operation costs including fuel costs and maintenance costs dominate the total cost of electricity supply. Fuel cost is the major consideration of the system operator in making generation schedules. In traditional generation dispatch, the operator's objective is to minimize the total fuel consumption or total fuel cost. It is necessary to understand the relationship between fuel consumptions and power generation outputs.

Heat rate and coal rate represent the amount of energy and coal, respectively, needed for one unit power output in a coal-fired steam power plant. The generation system composed of boiler, turbine, and generator is not a linear system. Heat rates/coal rates differ at different output levels. Temperatures and other factors of the boiler also affect coal consumptions. Usually, a nonlinear curve is used to represent the relationship of coal consumptions and power outputs. Most coal-fired power plants can provide scattered sample data for coal consumptions or coal rates at different power output levels. Polynomial curve fitting is used to obtain the curve for coal consumption versus power output. Figure 3.3 shows the sample data of coal consumptions of a 600 MW coal-fired power plant at different power outputs and the result of quadratic curve fitting.

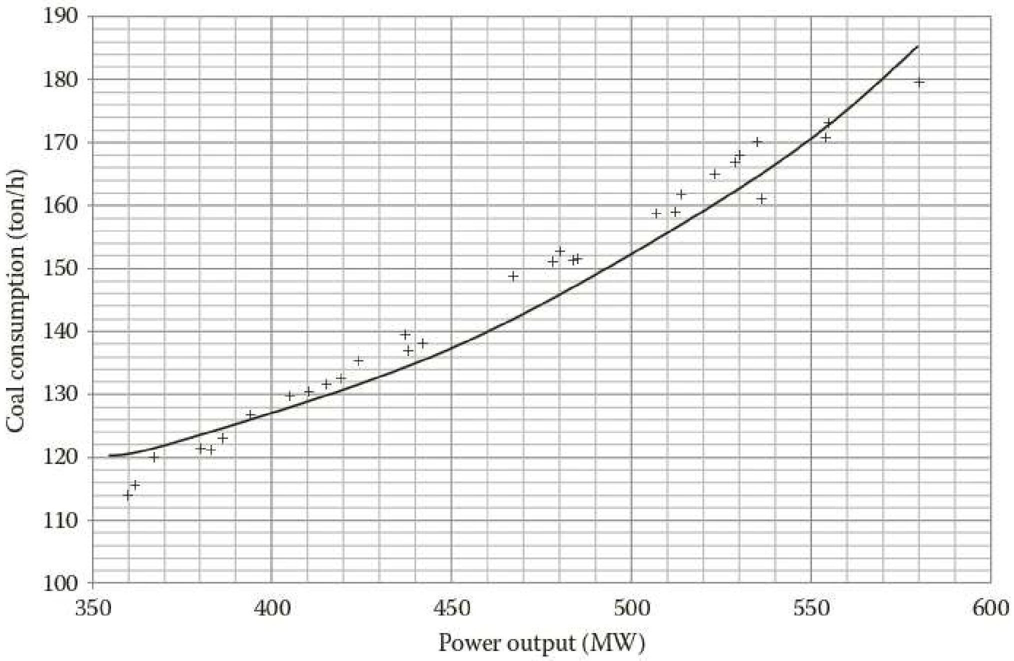

FIGURE 3.3 Sample data of coal consumption versus power output.

In power system economic operation, fuel consumption of a coal-fired generating unit is usually considered as a quadratic function of power output. The function can be obtained with measurement data through curve fitting. The fuel (coal) consumption of a generating unit can be expressed by

$$F = aP^2 + bP + c \tag{3.3}$$

where:

F is the amount of fuel consumed

P is the power output of the generating unit, and $P \in [P_{\min}, P_{\max}]$

a, b, and c are coefficients

For a generating unit, the input is fuel and the output is power generation. The amount of fuel (in lb. or kg) times the fuel heating value (in Btu/lb. or kJ/kg) is the heat (in Btu or kJ). The curve of fuel or heat input versus power output (solid line) is shown in Figure 3.4. Fuel rate or heat rate is calculated as $[F\ (\text{ton/h})/P\ (\text{MW})]$ or $[H\ (\text{Btu/h})/P\ (\text{MW})]$. In Figure 3.4, the slope of dashed line represents the fuel rate/heat rate at output level P. As the input–output curve for fuel/heat and power is convex and monotonically increasing, the minimum fuel rate/heat rate may be located at a point between $P_{\min}$ and $P_{\max}$,

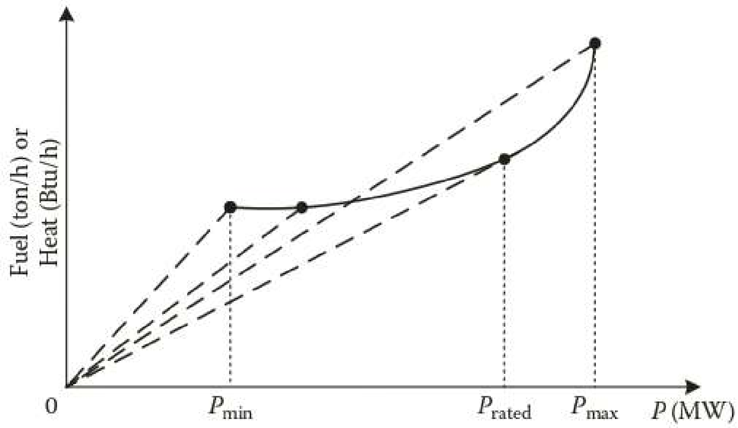

FIGURE 3.4 Input (fuel or heat) versus output (power) curve of generating unit.

which is P_{rated}, as shown in the figure. The point P_{rated} is the most economic operating point for the generating unit in a long-term operation. P_{rated} is called rated power output of the unit. The fuel rate/heat rate curve is shown in Figure 3.5.

Fuel consumption characteristics are different for different generating units. For coal-fired power plants, large capacity generating units usually have lower coal rates compared to small units. For example, the coal rate of a 1,000 MW generating unit at its full load is around 270 g/kWh, while the coal rate for a 100 MW generating unit could be as high as 330–360 g/kWh. Input (fuel)–output (power) curves could also be quite different for different power plants.

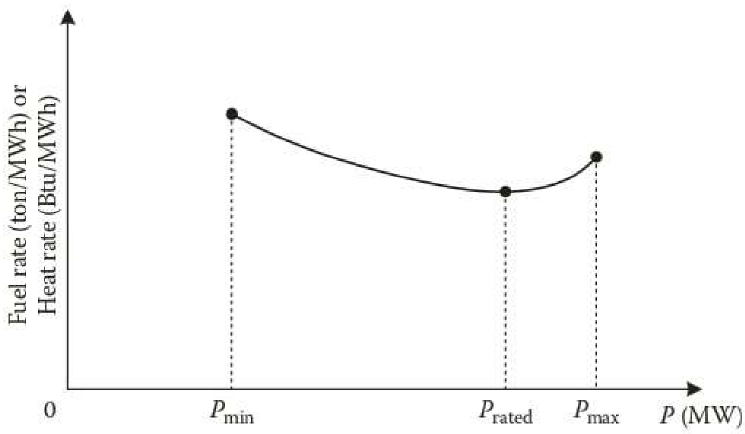

FIGURE 3.5 Fuel rate/heat rate versus power output curve of generating unit.

Example 3.1

Fuel consumption function of a real coal-fired generating unit is $F = aP^2 + bP + c$, where P is power output of the unit (in MW). F is the amount of fuel consumption (in ton standard coal equivalent per hour). The generation of the unit is limited between 250 and 500 MW. Coefficients a, b, and c are given as: $F = 0.0000618P^2 + 0.2599P + 9.4647$.

1. Derive the expression of fuel rate for the generating unit.
2. Find the rated operation point P_{rated} for the unit.
3. Calculate the unit fuel rate if the generation output is P_{rated}.

Solutions

1. Derive the expression of fuel rate for the generating unit.

   Fuel rate is expressed as $[F(\text{ton/h})/P(\text{MW})]$, so

$$\text{Fuel rate} = \frac{F\,(\text{ton/h})}{P(\text{MW})} = \frac{aP^2 + bP + c}{P} = aP + b + \frac{c}{P}$$

$$= \frac{0.0000618P^2 + 0.2599P + 9.4647}{P}$$

$$= 0.0000618P + 0.2599 + \frac{9.4647}{P}\ (\text{ton/MWh})$$

2. Find the rated operation point P_{rated} for the unit.

   The point P_{rated} is the most economic operation point for the generating unit. When generation output is equal to P_{rated}, the fuel rate is minimum, or the fuel rate should be equal to the marginal fuel consumption, as shown in Figure 3.4.

   The marginal fuel consumption is

$$\frac{\mathrm{d}F(P)}{\mathrm{d}P} = \frac{\mathrm{d}(aP^2 + bP + c)}{\mathrm{d}P} = 2aP + c$$

   At point P_{rated}, the marginal fuel consumption equals to the fuel rate, so

$$\frac{\mathrm{d}F(P)}{\mathrm{d}P}\Big|_{P=P_{\text{rated}}} = 2aP_{\text{rated}} + b \text{ equals to } \frac{F(P)}{P}\Big|_{P=P_{\text{rated}}}$$

$$= aP_{\text{rated}} + b + \frac{c}{P_{\text{rated}}}$$

   Then, $2aP_{\text{rated}} + b = aP_{\text{rated}} + b + \dfrac{c}{P_{\text{rated}}}$,

$$\Rightarrow aP_{\text{rated}} = \frac{c}{P_{\text{rated}}} \text{ and } P_{\text{rated}}^2 = \frac{c}{a}$$

$$\text{So, } P_{\text{rated}} = \sqrt{\frac{c}{a}}$$

Coefficients of the generating units are $a = 0.0000618$ and $c = 9.4647$. Then, we have

$$P_{\text{rated}} = \sqrt{\frac{c}{a}} = \sqrt{\frac{9.4647}{0.0000618}} = 391.34 \text{ MW}$$

It shows that the rated operation point P_{rated} is determined by coefficients a and c if the fuel consumption function is assumed to be a quadratic function.

3. Calculate the unit fuel rate if the generation output is P_{rated}.

   The fuel rate at generation output P_{rated} is

$$\frac{F(P)}{P}\Big|_{P=P_{\text{rated}}} = aP_{\text{rated}} + b + \frac{c}{P_{\text{rated}}}$$

$$= 0.0000618 \times 391.34 + 0.2599 + \frac{9.4647}{391.34}$$

$$= 0.0242 + 0.2599 + 0.0242$$

$$= 0.3083 \text{ ton/MWh}$$

$$= 308.3 \text{ g/kWh}$$

3.2.2 Gas-fired power plants

Combustion gas turbines using natural gas as the fuel are complementary to coal-fired steam turbines. The power output of gas turbine can be easily adjusted to follow quick load changes. The capital cost of a gas-fired power plant is much lower compared to that of a coal-fired power plant, but the fuel, usually natural gas, is relatively expensive. Many gas turbine-based power plants are designed to be easily switched to using oil as fuel, when the price of natural gas becomes high. Which type of power plant is more economical depends on the total fuel cost over the lifetime of the power plant. From carbon dioxide emission viewpoints, natural gas is more environmental friendly than coal. It is estimated that for the same amount of power output, the carbon dioxide produced by a gas-fired power plant is around half of that produced by a coal-fired power plant.

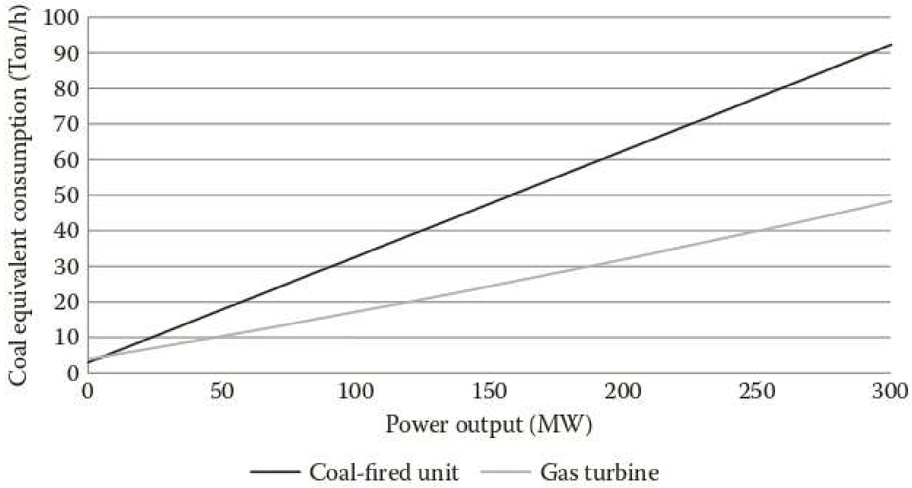

FIGURE 3.6 Coal equivalent consumption for coal-fired unit and gas turbine.

Gas turbines consume natural gas for power generation. The amount of natural gas generating 1 MW of power can be converted to the amount of coal equivalent. So, it is easier to compare fuel consumptions of gas turbines with those of coal-fired power plants. Similar applies to oil-fired turbines. The heat rate of 1 ton of coal equivalent is corresponding to 4.79 barrels of oil equivalent. One cubic feet (0.0283 cubic meter) of natural gas equals to 0.036 kg of coal equivalent, so 1 cubic meter natural gas is 1.272 kg coal equivalent.

In Figure 3.6, the fuel consumptions of a coal-fired power unit and a gas turbine are presented, where the natural gas consumption of the gas turbine has been converted to coal equivalent consumption. The coal equivalent consumptions at different power output levels shown in the figure are obtained from practical operation data. The figure shows that the gas turbine consumes less coal equivalent compared to the coal-fired generating unit. In other words, generating the same amount of power produces lower carbon emission from gas turbine.

Example 3.2

The coal equivalent consumption of a real gas turbine is $F = aP^2 + bP + c$, where P is in MW and F is in tons standard coal equivalent per hour. Coefficients a, b, and c are given as: $F = 0.0000209P^2 + 0.1680P + 16.6390$.

1. Find the rated power output P_{rated} for the unit.
2. Calculate the unit fuel rate if the gas turbine is operated at rated power level.

Solutions

1. Find the rated power output P_{rated} for the unit.

   According to the results obtained in Example 3.1, $P_{\text{rated}} = \sqrt{c/a}$. For the coal equivalent consumption function in this example, coefficients $a = 0.0000209$ and $c = 16.6390$, so we have $P_{\text{rated}} = \sqrt{c/a} = \sqrt{16.6390/0.0000209} = 892.25$ MW.

   The rated output level for the gas turbine is 892.25 MW.

2. Calculate the unit fuel rate if the gas turbine is operated at rated power output level.

   The fuel rate at P_{rated} is

$$\frac{F(P)}{P}\Big|_{P=P_{\text{rated}}} = aP_{\text{rated}} + b + \frac{c}{P_{\text{rated}}}$$

$$= 0.0000209 \times 892.25 + 0.1680 + \frac{16.639}{892.25}$$

$$= 0.0186 + 0.1680 + 0.0186$$

$$= 0.2052 \text{ ton/MWh}$$

$$= 205.2 \text{ g/kWh}$$

The results show that the fuel consumptions (in standard coal equivalent per kWh) for a gas turbine for generating one unit electricity is lower than that for a coal-fired generating unit.

3.2.3 Hydro power stations

Hydro power is one of the traditional generation resources. Moreover, it is a renewable energy resource among traditional generations. Around 20% of electricity in the world is produced by hydro power, which is cheaper and cleaner than thermal power plants. Large hydro power stations with dams are only part of the hydro power. Enormous dispersed small hydro power are valuable electricity generation resources, especially for remote areas.

Power generation of run-of-the-river hydro power relies on the water flow in the river and the weather. The generation from this type of hydro power is uncontrollable. The water reservoir of hydro power with dams performs storage function that makes power generation of this type of hydro power controllable. In general, hydro power generation subjects to water in the river or dam, rain cycle, and hydrological cycle.

In the power system operation, hydro power is a recommended generation resource due to its cheaper operation cost. The water used for power generation costs almost nothing.

Hydro turbines can quickly respond to load changes, so system operators prefer to use hydro power for frequency control to achieve real-time power balancing. Most of the time, power generations from hydro power stations are controllable, except the situation that water in the river or dam is insufficient due to hydrological cycles.

Similar to other renewable energy sources, construction cost dominates the generation cost of hydro power. The construction cost of a hydro power plant can be very high, and the operation cost is relatively low due to low fuel cost. The investment cost of hydro power plant varies for different countries and different projects. Generally, it is within the range of $700/kW to $3,500/kW. The loan is expected to be paid back with the revenue of hydro power generation. The payback period of the commercial loan for a power plant is usually 20–30 years. A hydro power station usually can generate electricity for 50 years or even longer. Accounting for the loan payback in the first 20–30 years, hydro power plant can generate power at a cost ranging from $0.04/kWh to $0.08/kWh, depending on projects. After the loan is paid, the generation costs drop significantly and could be within the range $0.01/kWh to $0.04/kWh. To calculate the cost of hydro power generation, both fixed cost and variable cost are counted. Fixed cost is mainly investment and its interests, insurance, salaries, and so on.

3.2.4 Nuclear power stations

Nuclear power is considered as a clean energy, as it produces no carbon emission when generating electricity. On the other hand, it is the most controversial power generation. The generation principle of nuclear power generation is the same as thermal power plants, except that the water is heated by nuclear reaction instead of fossil fuel. The fuel costs including the disposal cost of spent fuels are lower than that of coal-fired power plants.

The output of a nuclear power generating unit is usually very stable once it is started up. This is due to the limitation of nuclear reactor, and operators prefer not to change nuclear generation outputs too frequently. In many power systems, it is very common that nuclear power stations serve as baseload generators, which have the highest operating hours per year.

3.2.5 Wind power and solar energy

Wind power is one of the most promising renewable energy generation technologies developed in recent decades. The growth of wind power installed capacity has increased significantly all over the world. The benefits of wind power are no fuel cost and no carbon emission. Wind power is competitive in this way. However, investment cost of wind power is relatively high compared to traditional fossil-fuel power plants and hydro power. If considering the low annual operation hours due to uncontrollable wind resources, the average cost of generating 1 MW electricity by wind power in most cases is much higher than that of traditional power stations. Without economic incentives, it is hard to convince investors to invest in wind power. In many countries, new renewable energy projects are greatly facilitated by the renewable energy policies as well as renewable energy subsidies by governments. Carbon emission quotas and emission trading systems provide market incentives for renewable energy investment.

Solar energy is another promising renewable energy generation resource. Photovoltaic panels can directly convert solar energy to electricity. However, the conversion efficiency is only around 10% with current technologies. Although sunshine does not cost any money, the investment cost of photovoltaic panels is very high, even much higher than that of wind power. Government subsidies are necessary for solar energy to compete with wind power and other renewable energy.

In power system operations, uncontrollable renewable energy generations are forecasted in advance, just as load forecast. In generation scheduling models, forecasted renewable energy generations are used. In real time, actual generations from renewable energy resources deviate from the forecasted values. These deviated portions will be compensated by reserves and frequency regulation services. If the total renewable energy capacity in a system is so high, existing reserves might not be enough to compensate the deviations of renewable energy and loads. In this case, the patterns of power system operation and dispatch need to be adjusted to accommodate the high penetration of renewable energy generation.

3.3 Generation planning

3.3.1 Conventional generation planning

In traditionally vertically integrated power systems, power companies are responsible for generation planning, transmission planning, generation scheduling, and system real-time operation. Before late 1980s, computers were not widely applied in power system operations, and optimization technologies were not maturely developed for power system planning and operation. Engineers use some straightforward methods to manually obtain suitable generation mixtures for the system, as well as to manually minimize the cost of generation scheduling. Later, even after computer systems and various mathematic algorithms had been well developed for power system operations, the principle of generation planning and the way of generation scheduling and dispatch were still based on the traditional methods. Traditionally, each generating unit is assigned a certain number of operation hours per year. The number of annual operation hours depends on the type of the generating unit. For example, nuclear power is usually ON most of the time of the year except for maintenance. Its operation hours per year could be around 8,000 hours out of 8,760 hours per year. Large hydro power stations serving baseloads usually operate for 6,000–8,000 hours per year. For coal-fired power plants, the annual operation hours are around 5,000–7,000 hours. Oil-fired and combined-cycle units are around 2,000–4,000 hours, depending on the available generation sources in the system. Wind power is subject to wind resources, its annual operation hour is around 2,000 hours.

As mentioned in Section 3.1, electricity load has its cycle. Baseload is always there during the whole year. Nuclear power plants and coal-fired power plants are baseload units that have longer annual operation hours. This is due to the running requirements of nuclear and coal-fired power plants and their low variable costs. For a nuclear unit, once it is switched on, it is better to maintain a stable power output for a long term until it needs maintenance. For a coal-fired power plant, once the unit is turned on, there is a minimum requirement of ON time. Once the unit is turned off, it is required to be off for at least a certain time before the next start. On the other hand, even the unit is instructed to start, it takes some hours to warm up the furnace before the unit can start generating electricity. So, nuclear power plants and coal-fired power plants are usually scheduled to serve baseloads with relatively stable power outputs. The other reason of adopting nuclear power and coal-fired power plants as baseload units is their cheaper operating costs/fuel costs in relation to the outputs. It is beneficial to run the units with cheaper operation costs (in \$/kWh) for a large number of hours in 1 year. It is well known that large-scale centralized thermal power plants (including nuclear and coal-fired power plants) can lower down the system average generation cost due to economy of scale. However, large thermal power plants are not that fast in changing their outputs. For a sudden load change, it may take some time (e.g., 5–15 minutes) for a large thermal unit to follow the load. If generation outputs cannot quickly follow load changes, it may result in system frequency fluctuation in real-time operation.

Within the daily load cycle of most systems, there are quick load increases in the morning and in the afternoon. In some systems, the load could decrease by 10%–20% within 20 minutes before lunch time during weekdays. The generating units with fast-responding abilities are necessary to be planned and installed in a power system. Hydro power, combined-cycle generating units, and gas turbines are the types of generation resources that could respond quickly to adjust their power outputs for load following and frequency control. Latest energy storage technologies, such as flywheel, battery, and compressed air energy storage have even better performances in load following and frequency control. These generating units in most power systems serve as peaking generators. Hydro power has fast response rate as well as low variable operation cost. It is suitable for both baseload and peak load.

To fulfil the requirements of continuous baseloads and fast-changing peak loads, different types of generating units need to be installed. Due to the fact that most baseload units have high capital costs and relatively low operating costs, and most peaking units have low capital costs and high operating costs, it is necessary to design the capacities of different types of generators at the planning stage. The mixture of generations and their capacities are planned by considering capital costs and operating costs of generators as well as the annual load profile. Screening curves and load duration curves were invented by engineers as a simple and straightforward economical method for system generation planning.

3.3.2 Screening curve

To assess the cost of electricity generation from a power plant, both fixed cost and variable cost need to be considered. Fixed costs are the expenses no matter the power plant is on or off. Once a power plant is built, the capital investment cost needs to be paid back during the payback period, for example, 30 or 50 years. The power plant also pays tax, insurance, and other fees. On the other hand, there are some fixed maintenance costs even if the power plant is not turned on. All of them together, the capital cost, tax, insurance, fixed operation, and maintenance costs compose the fixed cost. For power plants, due to their high investments, the capital cost dominates the fixed cost. Variable costs are the expenses resulting from electricity generation when power plants are actually running. Variable costs are mostly fuel costs and operation costs. They are related to the amount of generated electricity.

Depending on different types of generations, fixed costs and variable costs are quite different. For example, coal-fired power plants have high fixed costs and low variable costs, while gas turbines have relatively low fixed costs and high variable costs. Comparison of the costs of two generators is not straightforward. Screening curve is developed to obtain the optimum mix of different types of power plants. It is based on the annualized fixed costs and variable costs.

In generation planning, the total capital cost of investing in a power plant is annualized into yearly fixed cost per kW (in \$/yr-kW) for each year of the payback period. The variable cost (in \$/yr-kW) is the annual fuel cost (in \$/kWh) plus annual operation and maintenance (O&M) cost (in \$/kWh) corresponding to the amount of annual electricity generation of the power plant. To simplify, we make a few assumptions: (1) the power plant runs at rated power during the operation hours and (2) the fuel cost is assumed to be constant or the average fuel cost is used. The variable cost can be expressed as

$$\begin{aligned}\text{Variable cost (\$/yr-kW)} = (&\text{Fuel cost (\$/kWh)} \\ &+ \text{O\&M cost (\$/kWh)}) \\ &\times \text{annual operation hour (h/yr)}\end{aligned} \tag{3.4}$$

For example, the capital cost for a coal-fired power plant is \$1,800/kW and the annualized fixed cost is \$270/yr-kW. Fuel cost for the plant is around \$0.36/kWh, and the O&M cost is \$0.04/kWh. Then, the variable cost is calculated as,

$$\begin{aligned}\text{Variable cost} &= (\$0.036/\text{kWh} + \$0.004/\text{kWh}) \\ &\quad \times \text{annual operation hours (h/yr)} \\ &= \$0.04/\text{kWh} \times \text{annual operation hours (h/yr)}\end{aligned}$$

If the annual operation hour of the power plant is 7,000 hours, the variable cost is \$280/kW.

The total annual cost is fixed cost plus variable cost, which is

$$\begin{aligned}\text{Annual cost} = \$270/\text{yr-kW} + \$0.04/\text{kWh} \\ \times \text{annual operation hours (h/yr)}\end{aligned}$$

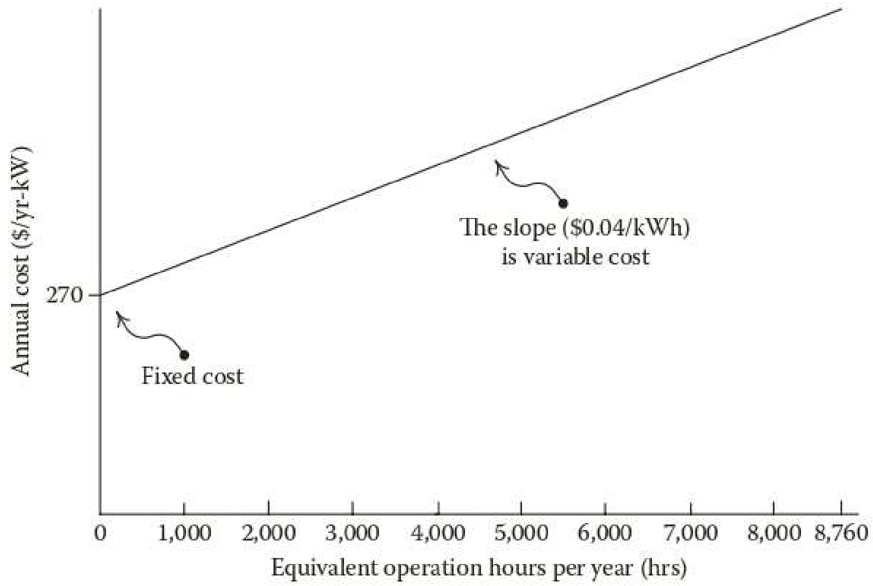

FIGURE 3.7 Fixed cost and variable cost for a generator.

The fixed cost and variable costs for the power plant are illustrated in Figure 3.7. The total cost of a power plant is a function of its operation hours.

The fixed cost and variable cost shown in Figure 3.7 are different from the generation cost functions shown in Figures 3.4 and 3.6, which represent the fuel consumptions at different power outputs (MW). The average fuel cost for rated power (P_{rated}), which is the slope of the dashed line obtained in Figure 3.4 could be used as the fuel cost in Equation 3.4.

For different types of generators, their fixed costs and variable costs are quite different. An example is used here to compare annual costs for a hydro power station, a coal-fired power plant and a gas turbine. Hydro power has the highest fixed cost, but its operation cost is the lowest due to its close-to-zero fuel cost. Gas turbine has the lowest fixed cost, but its operation cost is the highest due to its high fuel cost. Assume the annual cost for the power plants are as follows:

Hydro power station:	$450/yr-kW + $0.01/kWh × operation hours
Coal-fired power plant:	$270/yr-kW + $0.04/kWh × operation hours
Gas turbine:	$170/yr-kW + $0.08/kWh × operation hours

By comparing the annual cost curves of the previous three types of generations, the most economic generation type could

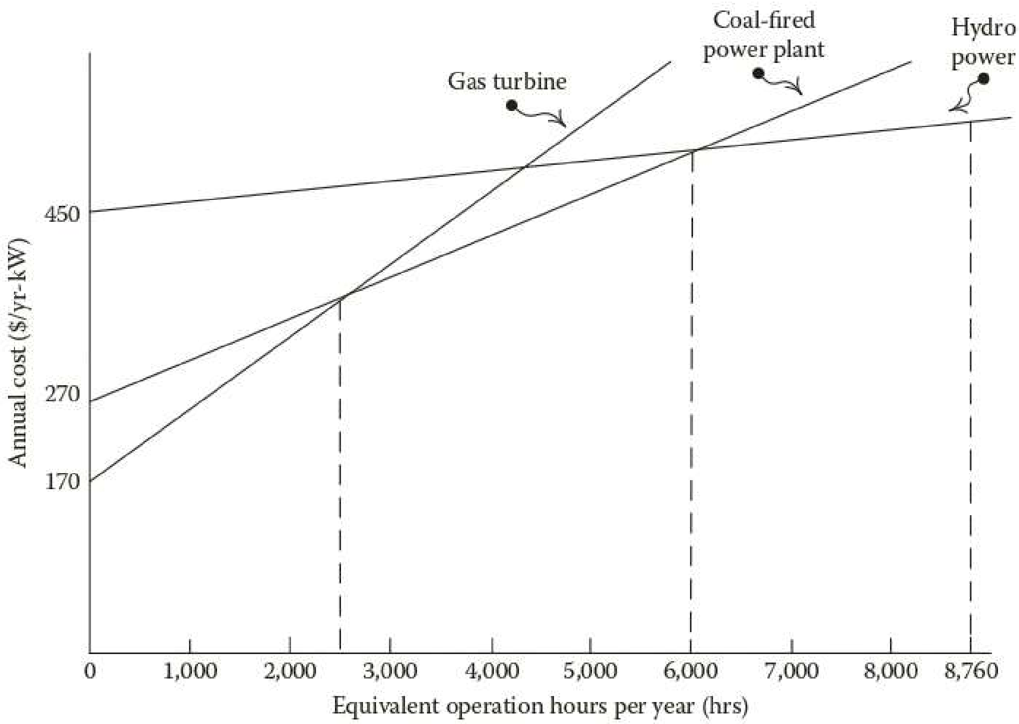

FIGURE 3.8 Screening curves for a hydro power plant, a coal-fired power plant, and a gas turbine.

be found for a specific operation hour. It is called screening curves, as shown in Figure 3.8.

From Figure 3.8, it could be observed that if an investor plans to invest in a power plant that operates less than 2,500 hours in 1 year, the most economical option is to invest in a gas turbine. If the power plant runs between 2,500 hours and 6,000 hours, a coal-fired power plant is the best option. If the power plant runs more than 6,000 hours per year, it is worth investing in a large-scale hydro power station, which has a high investment cost but a low operation cost.

Screening curves show which type of generator is the most economical option for different annual operation hours. However, the capacity of each type of generation cannot be determined unless the system load characteristics is given. If screening curves are used together with the system annual load duration curve, the optimal mix of various generators and their capacities can be obtained for generation planning.

3.3.3 Load duration curve

Electricity demand has its own cycle, as described in Section 3.1. The daily load curve and weekly load curve are provided in Figures 3.1 and 3.2. In the figures, the load for each hour of a day and a week is plotted in the chronological order. For residential and commercial load curves, peaking demands are in the morning and late afternoon. In Figure 3.9, the amount of load during each hour of a day is plotted with a column for 24 hours. If we rearrange the 24-hour loads of the day according to their volumes, we can obtain the load duration curve for the day, as shown in Figure 3.10. In the figure,

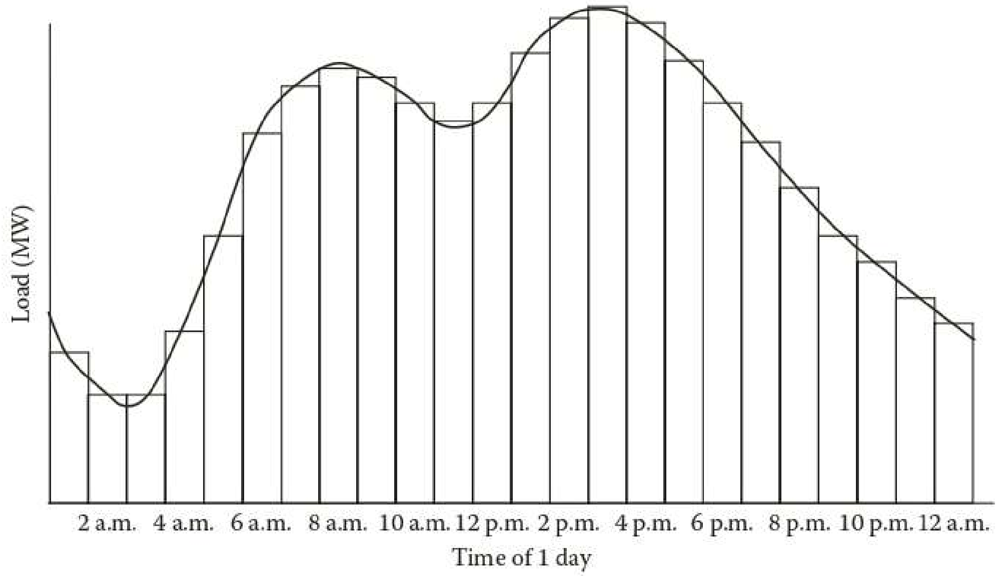

FIGURE 3.9 Daily load profile with hourly load representation.

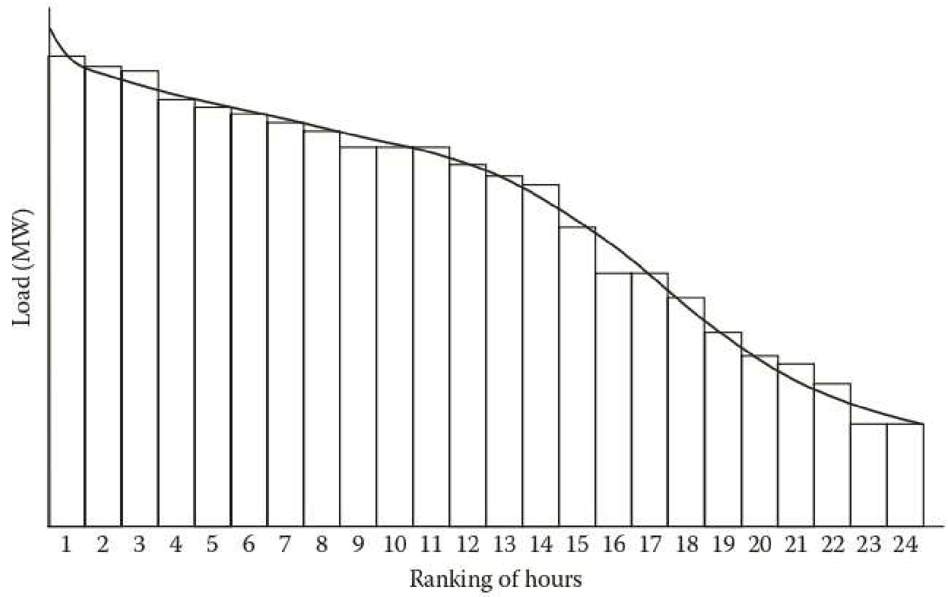

FIGURE 3.10 Daily load duration curve.

hourly loads are ranked according to their volumes, and the X-axis is the ranking of hours.

Figure 3.9 is the load profile of a day for 24 hours. The power (MW) of each hour is plotted in a time series, 1 a.m.,…, 11 a.m., 12 p.m., 1 p.m.,…, 11 p.m., 12 a.m. The highest load is during the time period of 3 p.m., the second highest is 2 p.m., and the third is during 4 p.m. The lowest loads are during time periods 2 a.m. and 3 a.m. By rearranging the loads from the highest to the lowest, the load at 3 p.m. is ranked as number 1 in Figure 3.10, load at 2 p.m. is ranked as number 2, and so on. In the end, loads at 2 a.m. and 3 a.m. are ranked as the last two, which are number 23 and 24. The rearranged load curve is the *load duration curve* of the day. If the ranking of the load profile is implemented for the whole year of 8,760 hours, the load duration curve for the whole year is obtained as shown in Figure 3.11. Here, it is assumed that the peak load of the system is 8,000 MW and the lowest load of the year is 2,800 MW. This load duration curve can be used for generation planning to decide the capacities of different types of generators based on their optimal annual operation hours obtained from screening curves. The process is described in Figure 3.11.

Let us continue to use the three types of generators, hydro power, coal-fired power plant, and gas turbines as the example to explain how to decide their capacities through screening curves and the load duration curve. From Figure 3.8, it has already been concluded that hydro power station has the best

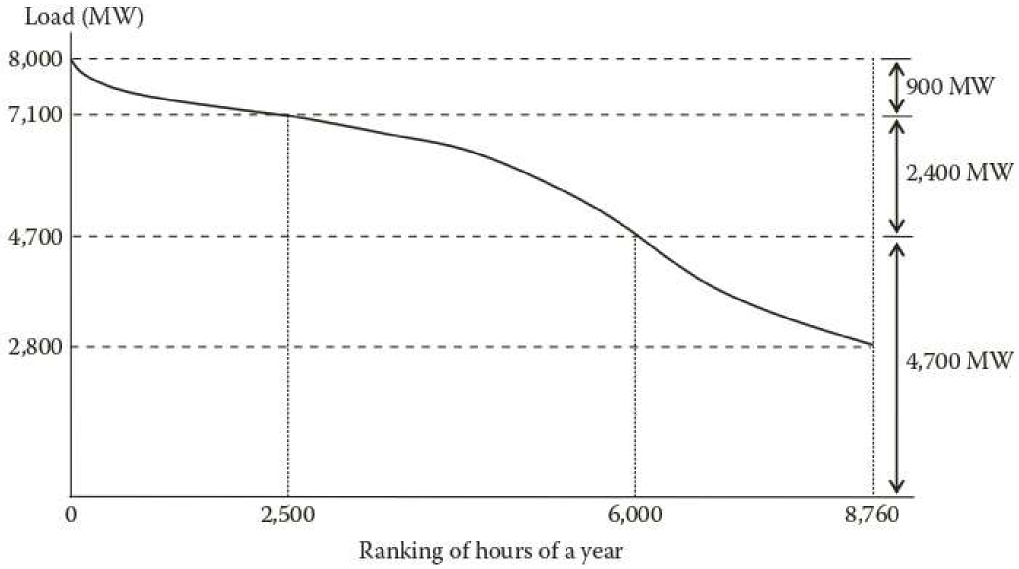

FIGURE 3.11 Load curve of a day.

economic effect (or lowest annual average generation cost) if it runs at rated power for more than 6,000 hours per year; and coal-fired power plant has the best economic effect for an operation hour ranged between 2,500 hours and 6,000 hours; and it is better if the gas turbine operates less than 2,500 hours per year. From Figure 3.11, it shows that for the whole year, there is always 2,800 MW electricity demand in the system. The minimum economic point for a hydro power station is to operate at least 6,000 hours per year. From the figure, if the hydro power has a generation ability of 4,700 MW, it can supply the load for at least 6,000 hours per year. So, the capacity of hydro power is determined as 4,700 MW. Similarly, the coal-fired power plant needs to operate at least 2,500 hours per year, which needs (7,100 MW – 4,700 MW) = 2,400 MW, of which the 4,700 MW is provided by the hydro power station. The rest capacity is provided by the gas turbine, which has the lowest investment cost and highest fuel cost. The capacity of the gas turbine is determined as (8,000 MW – 7,100 MW) = 900 MW. On the basis of the results obtained from Figure 3.11, with the optimal mix of generation capacities, we can observe that hydro power runs for most hours of the year to supply the baseload. Gas turbines only run during peak hours to supply the peak load, while coal-fired power plant in this system supplies the intermediate load. The effect can be easily observed, if we draw the operation hours of the three types of generators for a 24-hour period on a chronological daily load curve, as shown in Figure 3.12.

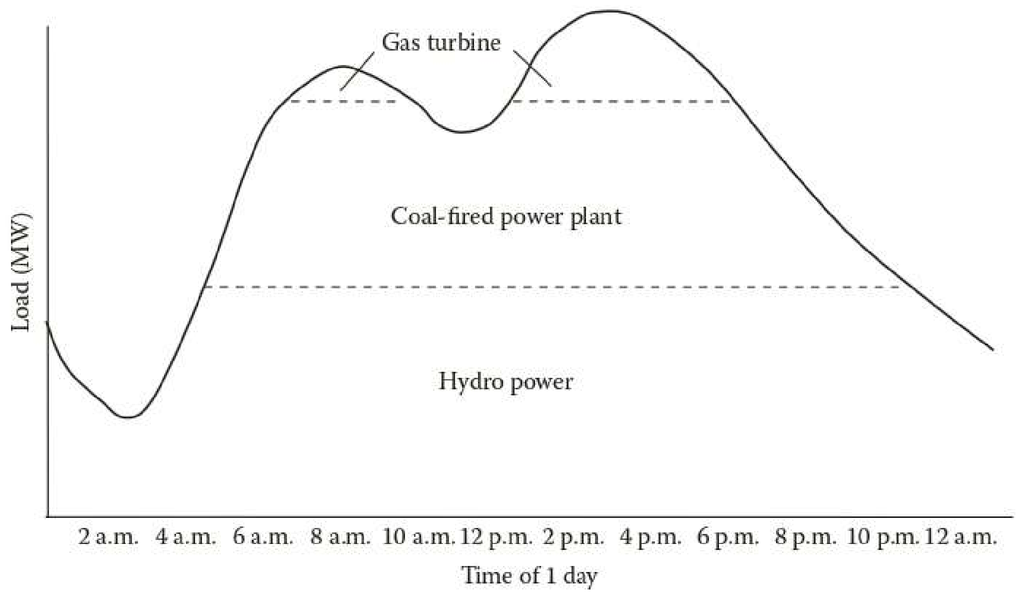

FIGURE 3.12 Load curve of a day.

It shows that the hydro power operates most of the time during the day except a few hours in the night. The gas turbine supplies only peak loads during morning peak and afternoon peak hours.

3.3.4 Generation planning and generation scheduling

Generation planning is a long-term optimization for power system economic operation. It determines the mixture of generation resources base on long-term load profiles and load growth. Generation planning coordinates with transmission expansion planning and provides technical optimization results for generation investment.

In short term, generation schedules are made to optimize unit generation outputs on a daily-base load profile. Generation scheduling optimizes fuel consumptions or generation costs with the given generation sources. Generation scheduling is for short-term power system economic operations. Economic dispatch, optimal power flow, and unit commitment, all are short-term optimization problems for procuring generation schedules. They will be introduced in Chapters 4 through 6.

3.4 Summary

The costs of generations are the major concern of power system planning and scheduling, while the costs are related to various factors. Generating units have different fixed costs and variable costs, mainly depending on the types of units. The fuel consumption rate of a specific generator changes at different output levels. Screening curves can be built for different types of generators according to their annualized fixed costs and average variable costs. Long-term generation planning can be determined by the screening curves and the forecasted load pattern of the system.

CHAPTER FOUR

Economic dispatch

4.1 Introduction

In a power system, utilities are responsible for planning and installing new power plants based on the forecasted load growth in the system. The available generation capacity in a system at all times should be higher than the peak load plus the required reserve margin due to reliability requirements. On the basis of the forecasted load profiles and maintenance schedules of generators, the system operator makes a schedule for unit commitment in advance. The ON/OFF schedule for generating units considers generator minimum ON/OFF time, reserve requirements, multi-time period coordination, as well as generator operation costs and startup costs, and so on. For a specific time period, say 1 hour, there are usually multiple generators being scheduled ON. The types of generators and their operation characteristics could be quite different. The system operator needs to make a generation dispatch schedule for all generators to satisfy the total electricity demand of each hour. Conventionally, fuel cost is the major concern of dispatching a generating unit. Units with less fuel costs are dispatched to generate more electricity. This simple economic principle is straightforward for generation dispatch; however, it is hard to be implemented accordingly due to nonlinear characteristics of power generations. Fuel consumptions and generation outputs are not in a linear relationship. To obtain the most economical solution for generation dispatch, optimization model is needed for the dispatch problem. In this chapter, we will introduce the principle and the mathematical model of generation economic dispatch, as well as its solution methods.

4.2 The problem of economic dispatch

The problem of generation economic dispatch was raised around 1930s. Conventionally, power companies own generating units, transmission and distribution facilities. Customers pay a fixed electricity rate for the energy supplied by power companies. The power company is responsible for running the whole system in a reliable and safe way. For the vertically integrated power system, the power company invests generators, transmission, and distribution systems and maintains them in a good operation conditions. The investment decisions of generators and transmission lines are made by solving the generation planning problem and transmission planning problem. The costs of generation and transmission investment are fixed costs. At the operation stage, variable costs are the only factor that can be adjusted for economic operation. For a system with thermal power plants, fuel costs dominate the operation variable cost. The system operator has realized that the total generation cost could be reduced significantly by coordinating the generation outputs of generators in the system. The problem of economic generation dispatch is modeled mathematically with optimization model.

The purpose of generation economic dispatch is to procure a generation schedule for each generating unit for a specific time period. The procured generation schedule should be economically optimized, which means that the total generation cost for all units based on this schedule is the minimum compared to any other generation schedules. The fundamental requirement of generation economic dispatch is that the total generation from all units should satisfy the system load of the time period, and the generation of each unit is within its operation upper and lower limits.

The objective and requirement of economic generation dispatch can be expressed using an optimization model. We can start from a simple model by ignoring the complicated nonlinear transmission network. As shown in Figure 4.1, the network is considered as a big node, and transmission losses in the network are ignored. The solution of economic dispatch is to find a generation schedule for power balance between power suppliers (generators) and demand (customers), just like any other commodities. For generation dispatch, the schedule is made usually for each hour of the coming day (24 hours). Generating units are dispatched by the system operator of their power output P_G (MW) for each hour. So, the total energy (MWh) supplied

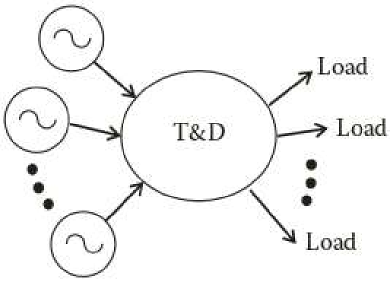

FIGURE 4.1 Power supply and demand.

by the unit within 1 hour is $Energy = P_G\,(\text{MW}) \times 1$ hour, which is in MWh. The generation schedule obtained from economic dispatch problem is the value of P_G for each generating unit for a specific hour.

We assume that the number of generating units in the system is N and the number of loads in the system is M. As the transmission network is considered as a node, the locations of the loads in the system are not important in this case. It is not necessary to identify individual load in the problem. We can use the total electricity demand of all customers as the system load, P_{Load}, which is known. The generation cost functions of generators are assumed to be known to the system operator in a centralized power system. The objective is to find the power generation schedule P_{Gi} for each generating unit i ($i = 1, 2, \ldots, N$), and the solution is the most economic one.

It has been discussed in Chapter 3 that the generation cost is a nonlinear function of power output. Here, a quadratic function is used to represent the cost function of generating unit i.

$$C_i(P_{Gi}) = aP_{Gi}^2 + bP_{Gi} + c \qquad \forall i = 1,\ldots,N \tag{4.1}$$

where $C_i(P_{Gi})$ is the generation cost function of unit i, and a, b, and c are coefficients.

The objective of generation economic dispatch is to minimize the total generation cost C_{Total}, which is the sum of the costs of all generating units. The objective function can be expressed as

$$\text{Min } C_{\text{Total}} = \sum_{i=1}^{N} C_i(P_{Gi}) \tag{4.2}$$

Power generation schedule P_{Gi} ($i = 1,\ldots, N$) in the objective function are control variables. They are subjected to power balance constraints (Equation 4.3), generation upper limit (Equation 4.4), and generation lower limit (Equation 4.5) of each unit.

$$\sum_{i=1}^{N} P_{Gi} = P_{\text{Load}} \tag{4.3}$$

$$P_{Gi} \le P_{Gi}^{\max} \tag{4.4}$$

$$P_{Gi}^{\min} \le P_{Gi} \tag{4.5}$$

The power balance Equation 4.3 ignores transmission network and losses. P_{Load} is the total system load. $P_{Gi}^{\max}$ is the maximum power output of generator i, or the capacity of the generator. $P_{Gi}^{\min}$ is the minimum generation requirement of generator i, due to turbine design.

The optimization model (Equations 4.2 through 4.5) is a nonlinear programming (NLP) problem. The constraints (Equations 4.4 through 4.5) are linear. The objective function (Equation 4.2) is a continuous nonlinear function, and it is convex. A typical method to solve the constrained optimization problem is to add constraints to the objective function by using Lagrange multipliers.

The constraint Equation 4.3 can be rewritten as

$$\phi = P_{\text{Load}} - \sum_{i=1}^{N} P_{Gi} \tag{4.6}$$

The Lagrange function L is to add the constraint function ϕ multiplied by an undetermined multiplier λ to the objective function C_{Total}, as shown in Equation 4.7.

$$\begin{aligned} L = C_{\text{Total}} + \lambda\phi &= \sum_{i=1}^{N} C_i\left(P_{Gi}\right) + \lambda\left(P_{\text{Load}} - \sum_{i=1}^{N} P_{Gi}\right) \\ &= \sum_{i=1}^{N}\left(C_i\left(P_{Gi}\right) - \lambda P_{Gi}\right) + \lambda P_{\text{Load}} \end{aligned} \tag{4.7}$$

In the Lagrange function, there are $N + 1$ variables, including N power outputs P_{Gi} and one Lagrange multiplier λ. The necessary condition for the extreme value of function L is when the first derivative of L with respect to each variable is equal to 0. For each independent variable P_{Gi} and λ, we have

$$\frac{\partial L}{\partial P_{Gi}} = \frac{\mathrm{d}C_i\left(P_{Gi}\right)}{\mathrm{d}P_{Gi}} - \lambda = 0 \qquad \forall i = 1,\ldots,N \tag{4.8}$$

and

$$\frac{\partial L}{\partial \lambda} = -\sum_{i=1}^{N} P_{Gi} + P_{\mathrm{Load}} = 0 \tag{4.9}$$

Then, the previous two equations can be simplified as a set of conditions as follows:

$$\begin{aligned} &\frac{\mathrm{d}C_i\left(P_{Gi}\right)}{\mathrm{d}P_{Gi}} = \lambda \qquad \forall i = 1,\ldots,N \\ &\sum_{i=1}^{N} P_{Gi} = P_{\mathrm{Load}} \end{aligned} \tag{4.10}$$

Equation 4.10 shows that the necessary condition for the minimum cost operation is that the first derivatives of generation cost functions of all generators are equal to a value λ and the sum of all generation outputs is equal to the total load. There are $N + 1$ equations and $N + 1$ variables in Equation 4.10. It is possible to solve the equations and obtain the values of P_{Gi} ($i = 1,\ldots, N$) and λ. The obtained P_{Gi} are the optimal generation schedule for all generators. In Economics, the first derivative of cost function with respect to the quantity of product is *incremental cost*. So, the obtained value λ is the incremental cost. In other words, when all generators are operated at the same incremental cost while satisfying the total load, the optimal generation schedule is obtained for minimum cost operation. This is economic dispatch solution, which is also called *equal-incremental cost* dispatch, or *equal-*λ dispatch.

The problem formulation of economic dispatch is based on the assumption that it is a centralized power dispatch. All generators are owned by the power company, and the system operator knows the cost function of each generator. Neglecting losses, the minimum cost solution is obtained when all generators are operated at the same incremental cost λ, which

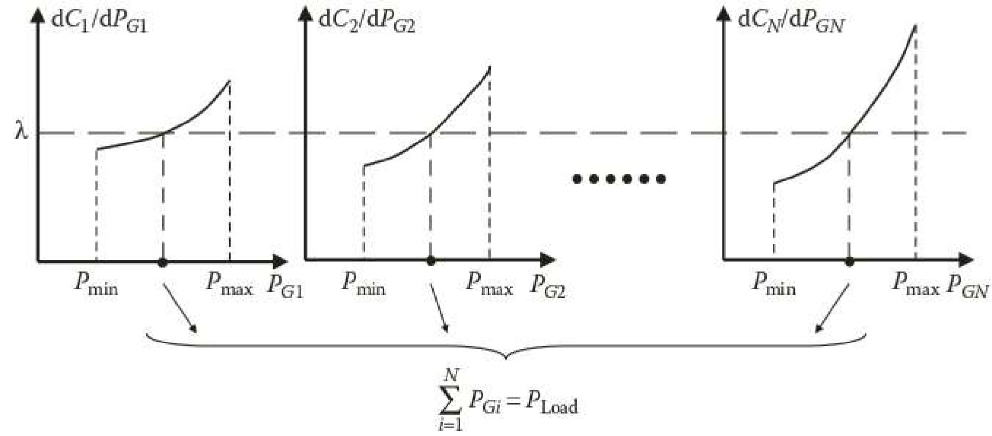

FIGURE 4.2 Economic dispatch results (within generation limits).

is also the marginal cost of the whole system. It is considered as the electricity price of the system.

The economic dispatch results without considering the inequality constraints (Equations 4.4 and 4.5) can be illustrated using Figure 4.2. In the figure, the dispatch results P_{Gi} all are within generation limits P_{Gi}^{Min} and P_{Gi}^{Max}.

The solutions of economic dispatch in Figure 4.2 are not constrained by the generation limits. In practical, the power output P_{Gi} of any generating unit should not exceed its rating P_{Gi}^{Max} while satisfying its minimum operation requirement P_{Gi}^{Min}. If the value of P_{Gi} obtained by Equation 4.10 for generator i exceeds P_{Gi}^{Max}, the solution for P_{Gi} should be set as P_{Gi}^{Max} ($P_{Gi} = P_{Gi}^{\text{Max}}$); similarly, if the obtained value of P_{Gi} is lower than P_{Gi}^{Min}, the solution for P_{Gi} should be set as P_{Gi}^{Min} ($P_{Gi} = P_{Gi}^{\text{Min}}$). Then, the equal-λ dispatch principle should be applied to the rest of generators to obtain the system λ while satisfying the remaining load. For the generator whose upper output limit is reached, the value $(\mathrm{d}C(P_{Gi}))/(\mathrm{d}P_{Gi})\big|P_{Gi} = P_{Gi}^{\text{Max}}$ of the generator should be lower than that of the system λ. Similarly, for the generator whose lower output limit is reached, the value $(\mathrm{d}C(P_{Gi}))/(\mathrm{d}P_{Gi})\big|P_{Gi} = P_{Gi}^{\text{Min}}$ of the generator should be higher than that of the system λ. This is shown in Figure 4.3.

With inequality constraints, the necessary conditions for an optimum can be found based on the complimentary slackness condition of KKT conditions. The results are shown in Equation 4.11.

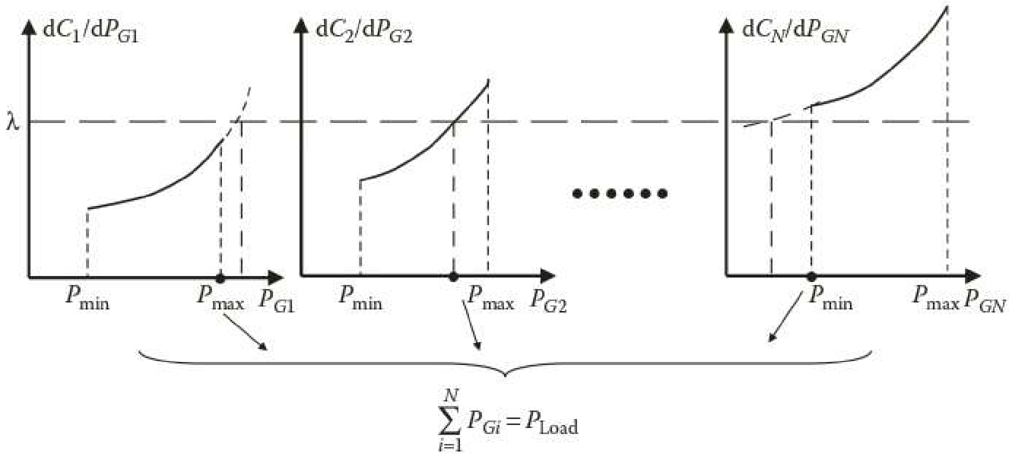

FIGURE 4.3 Economic dispatch results with inequality constraints.

$$
\begin{aligned}
&\frac{dC_i(P_{Gi})}{dP_{Gi}} = \lambda && \text{for } P_{Gi}^{\mathrm{Min}} < P_{Gi} < P_{Gi}^{\mathrm{Max}} \\
&\frac{dC_i(P_{Gi})}{dP_{Gi}} d \le \lambda && \text{for } P_{Gi} = P_{Gi}^{\mathrm{Max}} \\
&\frac{dC_i(P_{Gi})}{dP_{Gi}} d \ge \lambda && \text{for } P_{Gi} = P_{Gi}^{\mathrm{Min}} \\
&\sum_{i=1}^{N} P_{Gi} = P_{\mathrm{Load}}
\end{aligned}
\tag{4.11}
$$

Example 4.1

There are four generators in the system. Generator 1, 2, and 3 are coal-fired generators and generator 4 is a gas turbine. Their fuel consumption functions are given as follows:

Generator 1 $F(P_{G1}) = 0.148P_{G1}^2 + 199.4P_{G1} + 16425$
Generator 2 $F(P_{G2}) = 0.024P_{G2}^2 + 252.7P_{G2} + 16686$
Generator 3 $F(P_{G3}) = 0.136P_{G3}^2 + 206.3P_{G3} + 15433$
Generator 4 $F(P_{G4}) = 0.109P_{G4}^2 + 168.0P_{G4} + 16639$

where generation output P_{Gi} $(i = 1,\ldots, 4)$ is in MW, and fuel consumption is standard coal equivalent (in kg).

1. Find optimal generation schedule using economic dispatch neglecting losses if the system total load is 1,500 MW.

2. Repeat Question 1 if the cost of fuel (coal equivalent) of generator 4 is 1.25 times of the fuel cost of other coal-fired generators.
3. Find the total cost and system incremental cost given the coal price is \$80/tce (tce is ton of coal equivalent).

Solutions

1. Generation economic dispatch can be obtained by Equation 4.10. Incremental fuel consumption functions for the generators are

Generator 1 $\frac{dF(P_{G1})}{dP_{G1}} = 0.296P_{G1} + 199.4 = \lambda$

Generator 2 $\frac{dF(P_{G2})}{dP_{G2}} = 0.048P_{G2} + 252.7 = \lambda$

Generator 3 $\frac{dF(P_{G3})}{dP_{G3}} = 0.272P_{G3} + 206.3 = \lambda$

Generator 4 $\frac{dF(P_{G4})}{dP_{G4}} = 0.218P_{G4} + 168.0 = \lambda$

Power supply and demand balance is

$$P_{G1} + P_{G2} + P_{G3} + P_{G4} = P_{\text{Load}} = 1{,}500 \text{ MW}$$

There are in total five unknown variables in previous five equations. The equations can be solved as follows:

$$P_{G1} = 3.378\lambda - 673.65$$

$$P_{G2} = 20.833\lambda - 5264.58$$

$$P_{G3} = 3.676\lambda - 758.46$$

$$P_{G4} = 4.587\lambda - 770.64$$

Summing up P_{Gi} $(i = 1,\ldots,4)$ as expressed earlier, we have

$$P_{G1} + P_{G2} + P_{G3} + P_{G4} = 1{,}500 \text{ MW}$$

$$32.474\lambda - 7467.33 = 1{,}500$$

$$32.474\lambda = 8967.33$$

$$\lambda = 276.1 \text{ (kg/MWh)}$$

Then generation output of each generator can be calculated as follows

$$P_{G1} = 3.378\lambda - 673.65 = 259.0 \text{ MW}$$

$$P_{G2} = 20.833\lambda - 5264.58 = 487.4 \text{ MW}$$

$$P_{G3} = 3.676\lambda - 758.46 = 256.5 \text{ MW}$$

$$P_{G4} = 4.587\lambda - 770.64 = 495.9 \text{ MW}$$

This is the optimal generation dispatch results. Generators 2 and 4 consume relatively less fuels, so their generation outputs are higher in the economic dispatch result.

2. We assume that the coal price for coal-fired generators is ρ (in \$/kg). If the fuel cost of generator 4 is 1.25 times that of other coal-fired generators, its fuel price is 1.25ρ(in \$/kg). The cost function of each generator $C(P_{Gi})$ can be written as

Generator 1 $C(P_{G1}) = \rho(0.148P_{G1}^2 + 199.4P_{G1} + 16425)$
Generator 2 $C(P_{G2}) = \rho(0.024P_{G2}^2 + 252.7P_{G2} + 16686)$
Generator 3 $C(P_{G3}) = \rho(0.136P_{G3}^2 + 206.3P_{G3} + 15433)$
Generator 4 $C(P_{G4}) = 1.25\rho(0.109P_{G4}^2 + 168.0P_{G4} + 16639)$

Incremental cost functions for the generators are

Generator 1 $\dfrac{dC(P_{G1})}{dP_{G1}} = \rho(0.296P_{G1} + 199.4) = \lambda$

Generator 2 $\dfrac{dC(P_{G2})}{dP_{G2}} = \rho(0.048P_{G2} + 252.7) = \lambda$

Generator 3 $\dfrac{dC(P_{G3})}{dP_{G3}} = \rho(0.272P_{G3} + 206.3) = \lambda$

Generator 4 $\dfrac{dC(P_{G4})}{dP_{G4}} = 1.25\rho(0.218P_{G4} + 168.0) = \lambda,$

or $\rho(0.273P_{G4} + 210.0) = \lambda$

So, P_{Gi} $(i = 1,\ldots,4)$ can be expressed as

$$P_{G1} = 3.378\frac{\lambda}{\rho} - 673.65$$

$$P_{G2} = 20.833\frac{\lambda}{\rho} - 5264.58$$

$$P_{G3} = 3.676\frac{\lambda}{\rho} - 758.46$$

$$P_{G4} = 3.670\frac{\lambda}{\rho} - 770.64$$

Sum them up, it will be

$$P_{G1} + P_{G2} + P_{G3} + P_{G4} = 1{,}500 \text{ MW}$$

$$31.557\frac{\lambda}{\rho} - 7467.33 = 1{,}500$$

$$31.557\frac{\lambda}{\rho} = 8967.33$$

$$\lambda = 284.16\,(\text{kg/MWh}) \times \rho$$

The incremental coal consumption is 284.16 kg/MWh, and the incremental cost is 284.16 kg/MWh times coal price ρ. Then, generation schedules for four generators are

$$P_{G1} = 3.378 \times 284.16 - 673.65 = 286.2 \text{ MW}$$

$$P_{G2} = 20.833 \times 284.16 - 5264.58 = 655.3 \text{ MW}$$

$$P_{G3} = 3.676 \times 284.16 - 758.46 = 286.3 \text{ MW}$$

$$P_{G4} = 3.670 \times 284.16 - 770.64 = 272.2 \text{ MW}$$

The solutions show that the generation output of generator 4 is reduced due to its higher fuel cost. Other generators need to generate more electricity at a higher coal rate.

3. Given coal price as $\rho = \$80$ ton $= \$0.08$/kg, then

$$\lambda = 284.16 \text{ kg/MWh} \times \$0.08/\text{kg}$$
$$= \$22.73/\text{MWh}$$

The incremental cost is \$22.73/MWh.

The total generation cost is C_{Total}.

$$C_{\text{Total}} = C(P_{G1}) + C(P_{G2}) + C(P_{G3}) + C(P_{G4})$$
$$= \rho(0.148 \times 286.2^2 + 199.4 \times 286.2 + 16425)$$
$$+ \rho(0.024 \times 655.3^2 + 252.7 \times 655.3 + 16686)$$
$$+ \rho(0.136 \times 286.3^2 + 206.3 \times 286.3 + 15433)$$
$$+ 1.25\rho(0.109 \times 272.2^2 + 168.0 \times 272.2 + 16639)$$

$$C_{\text{Total}} = \rho(85616 + 192586 + 85644 + 88056)$$
$$= 451902 \text{ kg} \times \rho = 451.9 \text{ ton} \times 80\$/\text{ton}$$
$$= \$36,152$$

The total fuel cost is \$36,152 to generate 1,500 MW within 1 hour. The average cost is $\$36,152/1,500\text{MWh} = \$24/\text{MWh}$.

Example 4.2

There are four generators in the system. Their fuel consumption functions are the same as in Example 4.1.

Generator 1 $F(P_{G1}) = 0.148P_{G1}^2 + 199.4P_{G1} + 16,425$
Generator 2 $F(P_{G2}) = 0.024P_{G2}^2 + 252.7P_{G2} + 16,686$
Generator 3 $F(P_{G3}) = 0.136P_{G3}^2 + 206.3P_{G3} + 15,433$
Generator 4 $F(P_{G4}) = 0.109P_{G4}^2 + 168.0P_{G4} + 16,639$

In this example, generation limits of each generator are considered as follows:

Generator 1 $200\text{MW} \le P_{G1} \le 330\text{MW}$
Generator 2 $300\text{MW} \le P_{G2} \le 600\text{MW}$
Generator 3 $200\text{MW} \le P_{G3} \le 330\text{MW}$
Generator 4 $250\text{MW} \le P_{G4} \le 400\text{MW}$

Find the optimal generation schedule using economic dispatch neglecting losses and consider generation limits of generators. Total load in the system is 1,500 MW.

Solutions

First, the optimal solution without considering generation limits can be obtained. The results given in Question 1 of Example 4.1:

$$P_{G1} = 259\text{MW},\ P_{G2} = 487.4\text{MW},\ P_{G3} = 256.5\text{MW}, \text{ and}$$
$$P_{G4} = 495.9\text{MW}$$

Check the scheduled generations with the limits, we find that generations from generators 1, 2, and 3 are within the limits, while the generation of generator 4 is higher than its upper generation limit. So, we set the generation output of generator 4 as 400 MW, which is its upper limit, then dispatch the remaining demand requirement (1,500 MW – 400 MW = 1,100 MW) among the rest three generators. The generation economic dispatch for generators 1, 2, and 3 can be obtained by solving following equations:

$$\text{Generator 1} \quad \frac{dF(P_{G1})}{dP_{G1}} = 0.296P_{G1} + 199.4 = \lambda$$

$$\text{Generator 2} \quad \frac{dF(P_{G2})}{dP_{G2}} = 0.048P_{G2} + 252.7 = \lambda$$

$$\text{Generator 3} \quad \frac{dF(P_{G3})}{dP_{G3}} = 0.272P_{G3} + 206.3 = \lambda$$

The total generation from three generators should be 1,100 MW neglecting losses.

$$P_{G1} + P_{G2} + P_{G3} = 1{,}100\,\text{MW}$$

The above-mentioned four equations are solved as follows:

$$P_{G1} = 3.378\lambda - 673.65$$

$$P_{G2} = 20.833\lambda - 5264.58$$

$$P_{G3} = 3.676\lambda - 758.46$$

Replacing P_{G1}, P_{G2}, and P_{G3} in $P_{G1}+P_{G2}+P_{G3}=1{,}100$ MW, we have

$$27.887\lambda - 6696.69 = 1{,}100$$

$$27.887\lambda = 7796.69$$

$$\lambda = 279.58\,(\text{kg/MWh})$$

Then

$$P_{G1} = 3.378\lambda - 673.65 = 270.8\,\text{MW}$$

$$P_{G2} = 20.833\lambda - 5264.58 = 559.9\,\text{MW}$$

$$P_{G3} = 3.676\lambda - 758.46 = 269.3\,\text{MW}$$

$$P_{G4} = 400\,\text{MW}$$

As generator 4 reaches its upper limit 400 MW, while other generators need to generate more to satisfy the system load requirement. The incremental fuel consumption for generators 1, 2, and 3 with economic dispatch in this example is 279.58 kg/MWh. It is a little bit higher than the incremental fuel consumption 276.1 kg/MWh obtained in Question 1 of Example 4.1. The reason is that generator 4 reaches its upper generation limit. Other generators need to generate at higher output levels with higher incremental fuel consumptions. This is consistent with the curves shown in Figure 4.3. In addition, the generation of generator 4 is limited by 400 MW. At this output level, its incremental fuel consumption λ is calculated as

$$P_{G4} = 4.587\lambda - 770.64 = 400 \text{ MW}$$

$$4.587\lambda = 1170.64$$

$$\lambda = 255.2 \text{ kg/MWh}$$

4.3 Economic dispatch problem considering losses

The economic dispatch problem discussed in Section 4.2 neglects transmission losses occurred in the power network. In practical, transmission losses depend on the network topology, line parameters, generator locations, and electricity loads. Accurate transmission losses can be calculated by solving the power flow equations with the full network model considered. The nonlinear power flow equations are solved with Newton Raphson method that is programmed with computer programs. Before the computer was developed and used for power system analysis, power engineers first developed with equal-λ method to obtain optimal generation schedules neglecting transmission losses. Then, transmission losses were considered in the economic dispatch problem by simply using a quadratic function to represent losses.

As shown in Figure 4.4, consider transmission losses, P_{Loss}, the power balance in the system is written as

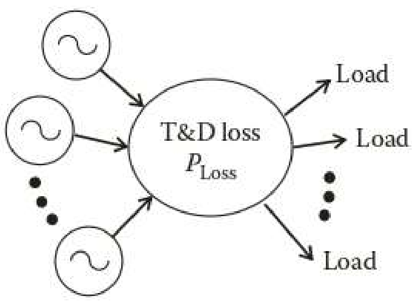

FIGURE 4.4 Power supply, transmission, and demand.

$$\sum_{i=1}^{N} P_{Gi} = P_{\text{Load}} + P_{\text{Loss}}, \text{ or } \phi = P_{\text{Load}} + P_{\text{Loss}} - \sum_{i=1}^{N} P_{Gi} = 0 \quad (4.12)$$

The economic dispatch problem including transmission losses is expressed by Equations 4.2, 4.4, 4.5, and 4.12. Using the same procedure, necessary conditions for the minimum cost dispatch is obtained. For the economic dispatch problem considering losses, we use symbol λ' as Lagrange multiplier. The Lagrange function is written as

$$\begin{aligned} L = C_{\text{Total}} + \lambda'\phi &= \sum_{i=1}^{N} C_i\left(P_{Gi}\right) + \lambda'\left(P_{\text{Load}} + P_{\text{Loss}} - \sum_{i=1}^{N} P_{Gi}\right) \\ &= \sum_{i=1}^{N}\left(C_i\left(P_{Gi}\right) - \lambda' P_{Gi}\right) + \lambda' P_{\text{Load}} + \lambda' P_{\text{Loss}} \end{aligned} \quad (4.13)$$

In the Lagrange function, P_{Loss} is not a constant number. It could be a function of generation outputs P_{Gi}. The first derivative of the Lagrange function with respect to variable P_{Gi} is derived as follows:

$$\begin{aligned} &\frac{\partial L}{\partial P_{Gi}} = \frac{dC_i\left(P_{Gi}\right)}{dP_{Gi}} - \lambda' + \lambda' \frac{\partial P_{\text{Loss}}}{\partial P_{Gi}} = 0 \qquad \forall i = 1,\ldots,N \\ &\frac{\mathrm{d}C_i\left(P_{Gi}\right)}{\mathrm{d}P_{Gi}} = \lambda'\left(1 - \frac{\partial P_{\text{Loss}}}{\partial P_{Gi}}\right) \end{aligned} \quad (4.14)$$

and

$$\frac{\partial L}{\partial \lambda'} = -\sum_{i=1}^{N} P_{Gi} + P_{\text{Load}} + P_{\text{Loss}} = 0 \quad (4.15)$$

Then, the above-mentioned two equations can be simplified as follows as a set of conditions:

$$\begin{aligned} &\frac{\mathrm{d}C_i\left(P_{Gi}\right)}{\mathrm{d}P_{Gi}} = \lambda'\left(1 - \frac{\partial P_{\text{Loss}}}{\partial P_{Gi}}\right) \qquad \forall i = 1,\ldots,N \\ &\sum_{i=1}^{N} P_{Gi} = P_{\text{Load}} + P_{\text{Loss}} \end{aligned} \quad (4.16)$$

Equation 4.16 are known as *coordination equations*. The item $(\partial P_{\text{Loss}}/\partial P_{Gi})$ is the incremental loss of generator i. The value of $(\partial P_{\text{Loss}}/\partial P_{Gi})$ represents the increased system losses caused by one unit generation increase from generator i. The factor $1/\left(1-\left[\partial P_{\text{Loss}}/\partial P_{Gi}\right]\right)$ is called *penalty factor*, pf_i.

$$pf_i = \frac{1}{\left(1-\left[\partial P_{\text{Loss}}/\partial P_{Gi}\right]\right)} \qquad \forall i = 1,\ldots,N$$

If penalty factor $pf_i > 1$, it means that increasing the generation from generator i will cause more losses in the system. This is a common case. If penalty factor $pf_i < 1$, it means that increasing the generation from generator i will reduce system losses, which is not so common. Coordination Equation 4.16 can be rewritten as follows:

$$\begin{aligned} & pf_i \times \frac{\mathrm{d}C_i\left(P_{Gi}\right)}{\mathrm{d}P_{Gi}} = \lambda' \qquad \forall i = 1,\ldots,N \\ & \sum_{i=1}^{N} P_{Gi} = P_{\text{Load}} + P_{\text{Loss}} \end{aligned} \tag{4.17}$$

For each generator, the incremental cost is corrected by a penalty factor that represents the impacts of generations on losses. The necessary condition of minimum cost operation with losses considered is that all generators are operated at the output levels such that their corrected incremental costs are equal to λ' and the total generation equal to the total load plus losses. The coordination equations and the results can be illustrated using Figure 4.5.

In the figure, the curves of generator incremental cost functions $\mathrm{d}C(P_{Gi})/\mathrm{d}P_{Gi}$ are shown with dashed lines. Equal-incremental cost principle is applied to obtain the generation schedule for minimum cost. Same as shown in Figure 4.2, the system incremental cost λ is obtained. By considering transmission losses, the incremental cost function is adjusted by the penalty factor. The curves of adjusted incremental cost functions are shown in solid lines in the figure. If the penalty factor is higher than 1, for example $pf_i = 1.05$ in the figure, the incremental cost function is moved up by 5%. In other words, more generations from this generator cause additional losses, so its incremental cost for power generation is increased after considering losses cause by itself. If the penalty factor is less

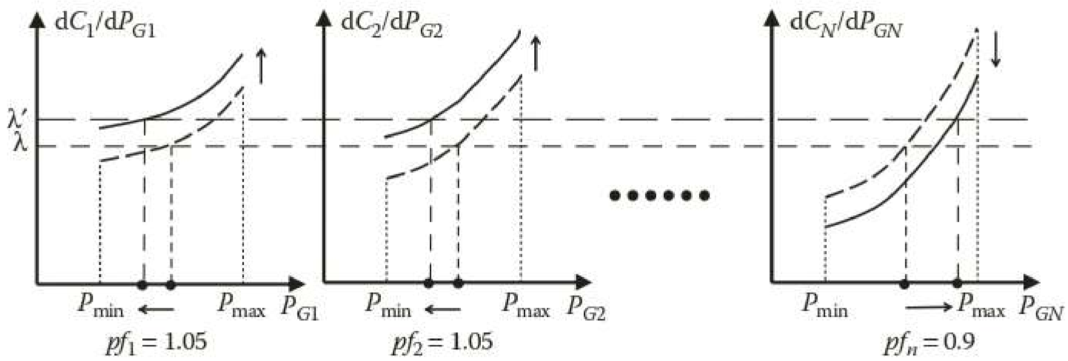

FIGURE 4.5 Economic dispatch results with transmission losses considered.

than 1 (although this seldom occurs in a power system), say $pf_i = 0.9$ in the figure, the incremental cost function is moved down by 10%. This means increasing the generation of the generator will reduce system losses and have a positive effect on cost-effectiveness. Of course, more generations should be encouraged from the generators with a penalty factor that is less than 1.

The solid curves in the figure represent adjusted incremental costs considering loss penalty factors. According to coordination Equation 4.17, the minimum cost solution considering losses is obtained when all adjusted incremental costs are equal to λ'. The results of λ' and the corresponding generation schedule for each generator are shown in Figure 4.5. It can be seen from the figure that compared to the economic dispatch results without considering losses, the generation output schedule for the generator with $pf_i > 1$ is moved leftward (output schedule is reduced) and the generation schedule for the generator with $pf_i > 1$ is moved rightward (output schedule is increased). The results are easy to understand. The generators causing more losses should have a reduction in generation outputs, and generators causing less losses should have an increase in power generation.

4.4 Economic dispatch with piecewise linear cost functions

In the economic dispatch problem discussed in previous sections, generation cost functions are assumed to be high-order nonlinear functions, for example, quadratic functions or cubic functions. In practical, generation companies prefer to use linear lines to represent generation cost curves. Multiple-segment

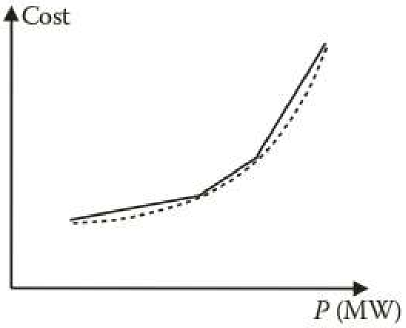

FIGURE 4.6 Piecewise linear generation cost function.

lines can closely represent a nonlinear function, as shown in Figure 4.6. The nonlinear generation cost curve, the dotted line in the figure, is approximately represented by three segments of straight lines.

If the generation cost is represented by a piecewise function instead of a continuous function, the necessary conditions obtained for minimum cost operation in previous sections are not suitable anymore. We can take the advantage of linear cost functions and solve the economic dispatch problem with simplified priority-list method. The solution method is explained in Figures 4.7 and 4.8. We assume there are three generators. Their cost curves are represented with piecewise linear functions, as shown in Figure 4.7. The first derivatives

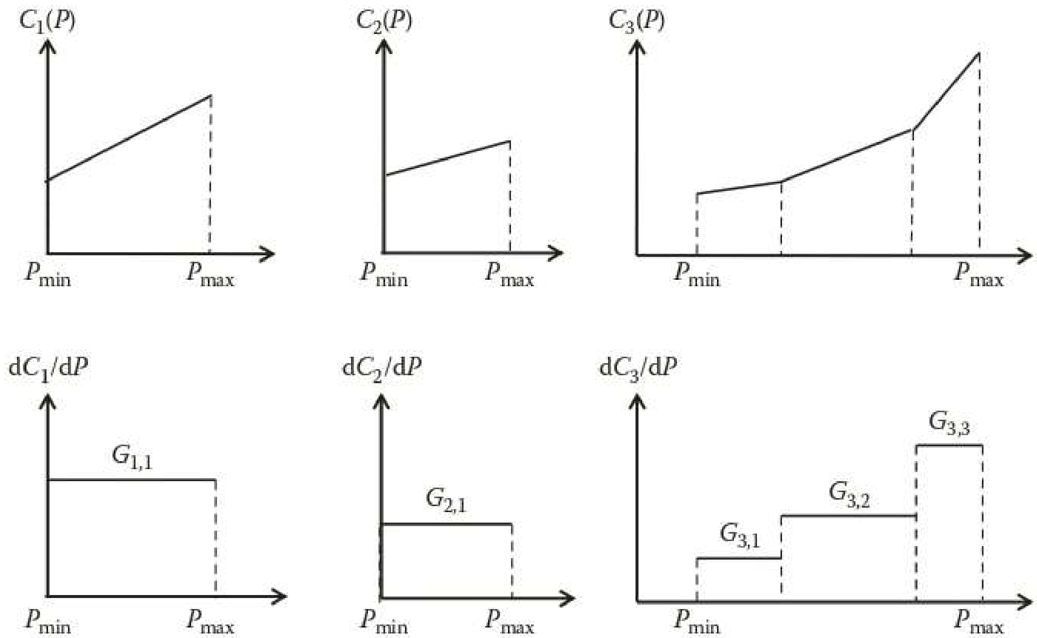

FIGURE 4.7 Piecewise linear generation cost curves and incremental costs.

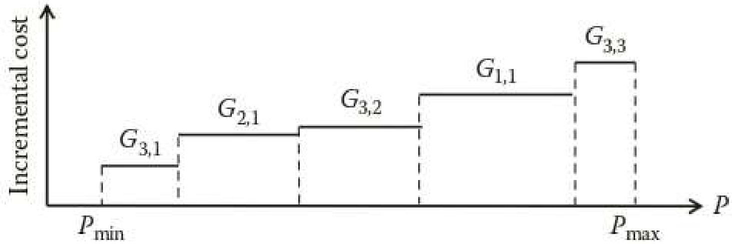

FIGURE 4.8 Incremental cost-based priority list.

of the cost functions are obtained and shown in the figure. Generators 1 and 2 use single segment linear lines to represent their generation costs. The first derivatives of the linear functions are constant numbers, which are incremental costs of the generators. Generator 3 uses three-segment linear lines to represent its generation cost. When generation output falls within different segments, the generation cost function has different slopes. So, the generator has different incremental costs at different generation output ranges. In Figure 4.7, the incremental costs of different segments of different generators are denoted with symbol $G_{i,j}$, where subscript i is the generator index, $i = 1$, 2, 3, and subscript j represents the number of line segments. Generators 1 and 2 have one segment, $j = 1$; generator 3 has three segments, $j = 1, 2, 3$, as indicated in the figure.

In practical, it is common that piecewise linear generation cost curves are provided instead of quadratic or cubic generation cost curves. Equal incremental cost criteria do not apply for this case, as incremental costs derived by piecewise linear cost curves are constant numbers. To obtain the generation schedule with the minimum generation cost, the system operator can setup a dispatch priority list based on the incremental cost of each generator. For example, the three generators shown in Figure 4.7 can be ranked in a priority list as shown in Figure 4.8. Generator 3 has the lowest incremental cost if it is dispatched for the first segment, so the first segment of generator 3, $G_{3,1}$, has the highest priority to be dispatched. The next lowest incremental cost is generator 2, which will have the second highest priority, and so on. Generators and their cost segments are arranged according to their priorities. Given the total system load, generators are selected according to the priority list until the system load is met. Then, the economic generation dispatch schedule is obtained.

4.5 Summary

Economic dispatch is a simplified power system economic operation method without considering full network. Economic dispatch has been commonly used in power systems since a long time. The equal incremental cost method and the coordination equation are efficient in procuring optimized generation schedules. It was developed before computer technologies were well developed to handle large systems. Optimal power flow is a more complicated version of economic dispatch by considering full network model using power flow equations.

Table 5.5 Optimal generation schedule for solving the AC power flow-based SCOPF model

Generator i	Generation schedule P_{Gi} (MW)
1	40
2	80
5	45.249
8	58.644
11	0
13	65.210

The optimal solutions for AC load flow-based SCOPF model are given in Table 5.5. The generation schedule shown in the table satisfies the network operation constraints for the base case and also satisfies the operation of any contingency topology if one of the lines is lost.

A comparison of the results obtained for SCOPF with the ACOPF results listed in Table 5.3 shows that the generation of generator at bus 11 is dispatched to be zero. The reason is that bus 11 has only one line connected to bus 9. This line is in the contingency set. To satisfy the requirement that if line 9–11 is disconnected, the generation schedule is still feasible to the rest system, the optimization results obtained is to have a zero output of generator 11. This is reasonable for SCOPF. To satisfy the demand for contingency scenarios, the more expensive generator at bus 5 is scheduled to generate more. The total cost obtained by SCOPF is \$10,184.51. It is higher than that of ACOPF without considering contingencies. This is the marginal cost for maintaining a higher security level.

5.7 Modified optimal power flow models for power system operations

In the OPF models we studied in previous sections, the objective function is to minimize the generation cost, which is a classical OPF model. In power system operations and planning, the objectives to optimizing the system operation vary for different purposes. The system operator could minimize generation cost, minimize power losses, minimize load shedding, or minimize reactive power. The system planner could optimize the cost for system expansion, optimize capacitor placement, and so on.

5.7.1 Minimize generation cost

In traditional power system economic operations, the objective for the system operator is to minimize the total generation cost as Equation 5.5:

$$\text{Min } C_{\text{Total}} = \sum_{i=1}^{N} C_i(P_{Gi})$$

The objective function is optimized subject to power flow Equation 5.6 and other operational limits Equations 5.7 through 5.10.

5.7.2 Minimize losses/total generation

Loss minimization is another commonly used objective function for power system operation. The generation cost of each unit is not considered. The goal of the OPF model is to minimize active power losses in the system operation as shown in Equation 5.37 subject to power flow equations and operation limits. As $\sum_{i=1}^{N} P_{Gi} = \sum_{i=1}^{N} P_{Di} + P_{\text{Loss}}$, if the load on each node is fixed, minimizing losses is equivalent to minimizing the total generation, as shown in Equation 5.38.

$$\text{Min } P_{\text{loss}} = \sum_{i=1}^{N} \sum_{\substack{k=1 \\ k \neq i}}^{N} G_{ik} \left(V_i^2 + V_k^2 - 2|V_i||V_k| \cos\theta_{ik} \right) \tag{5.37}$$

$$\text{Min } F = \sum_{i=1}^{N} P_{Gi} \tag{5.38}$$

5.7.3 Minimize generation adjustment

Besides minimizing total cost or total generation for generation scheduling, OPF can be used to minimize the generation adjustment. Generation schedules are made usually in advance, for example one-day ahead, based on forecasted load. In real time, the load profile varies from the forecasted load. OPF model can be used to obtain new generation schedules. In a market, the system operator may want to limit the deviation from the original generation schedule. So, the objective function of the OPF model can be written as the minimization of generation adjustment as shown in Equation 5.39, subject to power flow equations with real-time loads, and operation limits.

$$\text{Min } F = \sum_{i=1}^{N} \left| P_{Gi} - P_{Gi}^{\text{original}} \right| \tag{5.39}$$

where P_{Gi}^{original} is the original generation schedule obtained in advance. They are parameters in this optimization problem. P_{Gi} is the control variable to be optimized to minimize the total generation deviation from the original schedule.

5.7.4 Minimize load shedding

In some cases, the system operator needs to make load shedding schedules due to lack of generations, or emergent conditions. To limit the number of customers affected, the system operator tries to minimize the amount of load to be cut down. To obtain the optimal solution while considering load shedding priorities, the objective function for the OPF model is to minimize the total load shedding as in Equation 5.40, subject to power flow equations, operation limits, and constraints of load shedding priorities.

$$\text{Min } F = \sum_{i=1}^{N} P_{Di}^{\text{original}} - \sum_{i=1}^{N} P_{Di} \tag{5.40}$$

where P_{Di} is the load for each node i. It is the control variable in the OPF model. In the power flow equality constraints of the model, P_{Gi} is given as the scheduled available generation at node i and P_{Di} is the control variable to be determined as the optimal solution of the OPF model.

5.7.5 Optimization of reactive power

Reactive power provides voltage support in an AC power system and plays an important role in system stability. Reactive power is provided by generators, condensers, and VAr sources, such as capacitors, SVCs, STATCOM, and so on. As reactive power cannot be transmitted over long distances, only generators are not enough to provide reactive power required for a power grid. VAr sources need to be installed in the grid. Reactive power planning problem optimizes the installation and placement of VAr sources while satisfying power system operation requirements. The simplest reactive power optimization is to minimize the cost of VAr sources. Assume Q_{Ci} is the capacity of VAr resource installed on node i. The reactive power cost of VAr source i is usually a linear function represented by $C_{\text{VAr},i} = a_i + b_i Q_{Ci}$. Reactive power planning problem minimizes the total VAr cost Equation 5.41 subject to power flow equations and operation limits. This is a commonly used reactive power planning model.

$$\text{Min } F = \sum_{i=1}^{N} (a_i + b_i Q_{Ci}) \tag{5.41}$$

As reactive power support can affect transmission losses, the other commonly used reactive power planning model is to minimize both VAr cost and the cost of losses $C(P_{\text{loss}})$, as shown in Equation 5.42.

$$\text{Min } F = C(P_{\text{loss}}) + \sum_{i=1}^{N} (a_i + b_i Q_{Ci}) \tag{5.42}$$

Maintaining voltage deviations within operation limits is one of the tasks of the system operator. The goal of reactive power support could be minimizing the voltage deviations, subject to power flow equations and operation limits. The objective function can be written as Equation 5.43.

$$\text{Min } F = \sum_{i=1}^{N} \left(V_i^{\text{Max}} - V_i\right) \tag{5.43}$$

Generators and condensers are another reactive power providers. The capability of a generator to supply reactive power depends on its capacity and active power output. Reactive power support from generators can be continuously controlled. It is different from the discrete reactive power supply from VAr sources. Reactive power optimization problem also minimizes reactive power generation from generators. The objective function is to minimize the total reactive power generation from generators (Equation 5.44), or to minimize the total reactive power cost (Equation 5.45), subject to power flow equations and operation limits.

$$\text{Min } F = \sum_{i=1}^{N} Q_{Gi} \tag{5.44}$$

$$\text{Min } F = \sum_{i=1}^{N} C_i\left(Q_{Gi}\right) \tag{5.45}$$

In addition to the objective functions provided earlier, OPF models can be used for various scenarios in power system operation and planning to optimize different objectives.

5.8 Optimal power flow and unit commitment

In OPF problem, power system is optimized hourly for a given number of generators, N. In real power systems, the total number of generators is usually more than the number of generators that are running. Loads during peak hours are much higher than that in off-peak hours. Some generators are shut down during off-peak hours. Which generators should be switched on during peak hours and which generators should be shut down during off-peak hours are determined by the system operator using an optimization problem, called *unit commitment*. Once the units committed for each hour is determined, OPF is solved to obtain optimal generation schedules for each unit in the hour. Unit commitment problem is introduced in Chapter 6.

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CHAPTER FIVE

Optimal power flow

5.1 Introduction

Power system is a nonlinear system. Power losses occurring in the network during power transmission are affected by power grid parameters, locations of generators, generator outputs, and so on. In the conventional economic dispatch problem, transmission losses are ignored, in other words, power network is ignored and not shown in the dispatch problem. In fact, power network configurations, line parameters, generator locations, and generator outputs all affect power losses, as well as the economic effects of generation dispatch. In the economic dispatch problem considering losses introduced in Chapter 4, transmission losses are represented by a simplified function of generator outputs. Power network are still not considered in the optimization model, although power grid configurations and parameters significantly affect the distribution of power flow and active power losses.

The objective function of an economic dispatch problem is to minimize the total generation cost. In a conventional economic dispatch problem, the objective function is optimized subject to the simplified power balancing equation that the total power generation equals to the system load. This simplification significantly reduces the complexity of the optimization problem. To obtain a better economic dispatch solution that can accurately consider power network effects on optimal generation schedules, accurate power network model is needed to replace the simplified power balancing equation as the constraints of the optimization problem of economic dispatch.

Ohm's law shows the relationship of voltage and current of a simple circuit. For a complicated power network, the relationship of injected currents at the nodes and the voltages at the nodes can be represented via an admittance representation. Bus admittance matrix is used for accurate representation of a power network. Admittance matrix is widely applied in power system analysis in determining the network solutions. Bus impedance matrix is also a representation of network. It is more commonly used in fault analysis. Admittance matrix and impedance matrix for a network include the configuration and line parameter information of the network. In most situations, the network configuration and line parameters remain unchanged, so bus admittance matrix remains fixed. Changes in operation conditions are reflected by injected currents and voltages at the nodes. Generators and loads connected to the nodes inject power and withdraw power from the network. The product of bus voltage and injected current is the injected power. At any moment, the power injected in a node should be equal to the total power that flows from the node. In other words, the power input and output at a node is balanced in real time. Power balancing equations of all nodes can represent the operation conditions of a power grid as well as network configuration and line parameters. A set of power flow equations mathematically represents the full power network. If the full network model is included in the optimization problem of economic dispatch as power balancing constraints, the optimization results obtained for the problem can accurately consider power grid parameters. Applying power flow equations as full network model in the constraints of economic dispatch problem is called optimal power flow (OPF).

OPF has been widely applied in power system analysis and operation, especially in the economic related system analysis. OPF is solved every year for long-term power system planning. For daily operation, OPF is solved every day for daily generation scheduling and day ahead market; and it is solved every hour and every 5 minutes for hourly ahead market and power balancing market. OPF problem was first formulated in 1962. Due to the high nonlinearity of the problem, it is difficult to achieve a converged optimal solution in a short time.

Researchers are still searching for fast and robust solution algorithms for OPF. A good solution technique could save a huge amount of money for a power system. With the development of computing technologies, more system operation constraints can be modeled in the OPF problem to accurately reflect system operation conditions. In the electricity market, OPF model is solved for market settlement and market pricing.

5.2 Power flow formulations

5.2.1 Bus admittance matrix

Power network is a complicated system due to its meshed connections of power line components, (R, L, and C) and generation sources. The relationship of voltage and current cannot be represented simply using Ohm's law. Applying Kirchhoff's current law at nodes of a circuit is the basis for systematic formulation of a power network. For a system with N nodes, current injected to node i ($i = 1, \ldots, N$) is I_i and voltage at node k ($k = 1, \ldots, N$) is V_k. The relationships of currents and voltages at nodes can be represented in matrix form as

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1k} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2k} & \cdots & Y_{2N} \\ \vdots & \vdots & & \vdots & & \vdots \\ Y_{i1} & Y_{i2} & \cdots & Y_{ik} & \cdots & Y_{iN} \\ \vdots & \vdots & & \vdots & & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{Nk} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \\ \vdots \\ V_N \end{bmatrix}$$

The matrix is designated as **Y** and called *bus admittance matrix*. The node equations in matrix notation are expressed as $\mathbf{I} = \mathbf{YV}$, where **I** and **V** are column matrices and **Y** is a symmetrical matrix. For a network with N independent nodes, the current injected to node i is

$$I_i = \sum_{k=1}^{N} Y_{ik} V_k \qquad \forall i = 1, \ldots, N \tag{5.1}$$

The bus admittance matrix **Y** represents the electrical behavior of all network components. Matrix entries Y_{ik} is determined by components connected to node i. In this section, we provide the formulations of Y_{ik} in terms of admittances of all network components. The analytical approach to obtain the formulations is referred to Stevenson 1982 (Chapter 7).

A network is composed of branches connected between two nodes and components connected between a node and the ground. For example, if a line connects between node i and node k, its branch impedance is $z_{ik} = r_{ik} + jx_{ik}$, where r_{ik} is line resistance and x_{ik} is line reactance. Then, the branch admittance is y_{ik}, and ($y_{ik} = 1/z_{ik}$). For a component connected to node i, for example, a shunt capacitor or a shunt reactor, its branch admittance is y_{i0}, which is equal to the admittance of the component.

The diagonal element of bus admittance matrix, Y_{ii}, is self-admittance. Y_{ii} is calculated as the sum of branch admittances of the branches that connect to node i.

$$Y_{ii} = y_{i0} + \sum_{k=1}^{N} y_{ik} \quad \forall i = 1,\ldots,N$$

The off-diagonal element of bus admittance matrix Y_{ik} $(i \neq k)$ is calculated as the negative of the branch admittance between node i and node k.

$$Y_{ik} = -y_{ik} \quad \forall i = 1,\ldots,N \text{ and } i \neq k$$

As matrix $\mathbf{Y}$ is symmetric, off-diagonal element Y_{ki} equals to Y_{ik}, so $Y_{ik} = Y_{ki}$. Bus admittance matrix entries Y_{ik} $(i,k = 1,\ldots,N)$ are complex numbers. They are formulated as

$Y_{ik} = G_{ik} + jB_{ik} \quad \forall i,k = 1,\ldots,N$, where G_{ik} is *conductance* and B_{ik} is *susceptance.*

The number of nodes (or buses) in an interconnected power system is large. In practical, most buses in a power network connect not more than five other buses, and many of them connect only three buses. In the bus admittance matrix, a large number of entries are zero. For example, if the mth node connects three other nodes, among N entries of the mth row of the admittance matrix Y_{mk} $(k = 1,\ldots,N)$, only three entries plus the self-admittance Y_{mm} are nonzero. The rest $N - 4$ entries are zero. Consider a network with more than 1,000 buses, or a simplified configuration with more than 100 buses, the number of zero entries is large for bus admittance matrix. This is referred as the sparsity feature of bus admittance matrix. The bus admittance matrix for a real power network is a symmetric sparse matrix.

5.2.2 Power flow equations

In power system analysis, a power network is modeled by many nodes interconnected by branches. Generators, loads, and shunt components are connected to nodes as power sources inject or withdraw power from the power network. One-line diagram is used for representation of a power system.

A power system is composed of many nodes. Kirchhoff's current law applies to each node. Here, we first study the power balance of each node. The ith node of a N-node system is shown in Figure 5.1. S_{Gi} is the complex power injected to node i from the generator connected to the node, $S_{Gi} = P_{Gi} + jQ_{Gi}$. S_{Di} is the load withdrawn from node i, $S_{Di} = P_{Di} + jQ_{Di}$. S_{ik} is the complex power that flows from node i to node k, while $k = 1,\ldots,N$

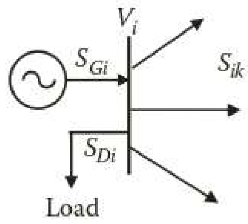

FIGURE 5.1 Power balance of node i.

and $k \neq i$. If we define S_i as the net power injection to node i, $S_i = S_{Gi} - S_{Di}$. The power injected into node i equals to the power flowing from node i. This power balance applies to any node $i = 1,\ldots,N$. Power balance for any node i can be formulated as

$$S_i = S_{Gi} - S_{Di} = \sum_{k=1}^{N} S_{ik} \quad \forall i = 1,\ldots,N$$

The voltage at node i is V_i. Complex power S_i at node i is calculated by $S_i = V_i I_i^*$, where I_i is the current injected into node i. The relationship of injected currents and node voltages is expressed by bus admittance matrix, as in Equation 5.1, so we have

$$\begin{aligned} S_i &= V_i I_i^* = V_i \left(\sum_{k=1}^{N} Y_{ik} V_k \right)^* \\ &= V_i \left(\sum_{k=1}^{N} Y_{ik}^* V_k^* \right) \quad \forall i = 1,\ldots,N \end{aligned} \tag{5.2}$$

Voltage V_i is a complex number. We use a polar representation $V_i = |V_i| e^{j\theta_i}$, where θ_i is phase angle of voltage V_i. If we use rectangular representation of admittance $Y_{ik} = G_{ik} + jB_{ik}$, then Equation 5.2 becomes

$$\begin{aligned} S_i &= V_i \left(\sum_{k=1}^{N} Y_{ik}^* V_k^* \right) \\ &= |V_i| e^{j\theta_i} \left(\sum_{k=1}^{N} (G_{ik} + jB_{ik})^* \left(|V_k| e^{j\theta_k} \right)^* \right) \\ &= |V_i| e^{j\theta_i} \left(\sum_{k=1}^{N} (G_{ik} - jB_{ik}) \left(|V_k| e^{-j\theta_k} \right) \right) \end{aligned}$$

$$= \sum_{k=1}^{N} |V_i| e^{j\theta_i} |V_k| e^{-j\theta_k} \left(G_{ik} - jB_{ik} \right)$$

$$= \sum_{k=1}^{N} |V_i||V_k| e^{j(\theta_i - \theta_k)} \left(G_{ik} - jB_{ik} \right) \quad \forall i = 1, \ldots, N$$

Define $\theta_{ik} = \theta_i - \theta_k$, then we have

$$\begin{aligned} S_i &= \sum_{k=1}^{N} |V_i||V_k| e^{j\theta_{ik}} (G_{ik} - jB_{ik}) \\ &= \sum_{k=1}^{N} |V_i||V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik}) \\ &= \sum_{k=1}^{N} |V_i||V_k| ((G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) \\ &\qquad + j(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik})) \qquad \forall i = 1, \ldots, N \end{aligned} \tag{5.3}$$

As complex power S_i is defined as $S_i = P_i + Q_i$, so we have

$$\begin{aligned} P_i &= \sum_{k=1}^{N} |V_i||V_k| \left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik} \right) \qquad \forall i = 1, \ldots, N \\ Q_i &= \sum_{k=1}^{N} |V_i||V_k| \left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik} \right) \qquad \forall i = 1, \ldots, N \end{aligned} \tag{5.4}$$

or

$$P_{Gi} - P_{Di} = \sum_{k=1}^{N} |V_i||V_k| \left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik} \right) \quad \forall i = 1, \ldots, N$$

$$Q_{Gi} - Q_{Di} = \sum_{k=1}^{N} |V_i||V_k| \left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik} \right) \quad \forall i = 1, \ldots, N$$

These are power flow equations for a power network. For each node, there are two equations. By solving all power flow equations of a network, power flow on each branch and voltage magnitude and phase angle of each node are obtained. Solution algorithms for power flow are referred to Stevenson 1982 (Chapter 8) and Bergen and Vittal 2000 (Chapter 10).

5.3 Optimal power flow modeling

Power flow equations include all branch parameters and network components, and nodal power balancing is formulated for each node of the system. In economic dispatch problem formulated in Chapter 4, power balancing is represented with a simplified equation without network parameters and generator locations. As power flow equations can mathematically represent the full network of a power system, we can apply power flow equations to economic dispatch problem. Then, the economic dispatch problem is to optimize system generation cost subject to power flow constraints of the full network. The network can be accurately considered in the optimization problem. Combining economic dispatch problem and power flow equations, the model we obtained is the OPF model.

5.3.1 Mathematical model of optimal power flow

In power system operation, the classical optimization problem is to minimize generation cost. The objective function of OPF problem is the same as economic dispatch problem to minimize the total generation cost C_{Total}, which is the sum of the costs of all generating units. The objective function can be expressed as

$$\text{Min } C_{\text{Total}} = \sum_{i=1}^{N} C_i(P_{Gi}) \tag{5.5}$$

where P_{Gi} ($I = 1,\ldots, N$) is generation schedule of the generating unit connected on node i and C_i is the cost function of the generating unit. The objective function is optimized subject to power flow equations for each node (Equation 5.6), and other operations limits (Equations 5.7 through 5.10).

Power flow constraints:

$$\begin{aligned} P_{Gi} - P_{Di} &= \sum_{k=1}^{N} |V_i||V_k|\left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) \quad \forall i = 1,\ldots,N \\ Q_{Gi} - Q_{Di} &= \sum_{k=1}^{N} |V_i||V_k|\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) \quad \forall i = 1,\ldots,N \end{aligned} \tag{5.6}$$

Generation limits:

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \quad \forall i = 1,\ldots,N \tag{5.7}$$

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \quad \forall i = 1,\ldots,N \tag{5.8}$$

Bus voltage limits:

$$V_i^{\min} \le V_i \le V_i^{\max} \quad \forall i = 1,\ldots,N \tag{5.9}$$

Transmission line limits:

$$\left|S_{ik}\right| \le S_{ik}^{\max} \quad \forall i,k = 1,\ldots,N \text{ and } i \ne k \tag{5.10}$$

where S_{ik} is the complex power that flows from node i to node k. The transmitted power on a line should be lower than its transmission capability limit.

The objective function (Equation 5.5), equality constraints (Equation 5.6), and inequality constraints (Equations 5.7 through 5.10) form the classical OPF problem that minimizes the cost of generation. In this optimization model, generator active power output P_{Gi} and reactive power output Q_{Gi} are control variables (or decision variables). For a P–V bus, generator voltage V_i is control variable and voltage phase angle θ_i is state variable. In power system operations, according to different purposes, other objective functions can be optimized. For example, for reactive power optimization problem, the objective function is to minimize the total reactive power generated by generators. In some situations, the system operator uses OPF to minimize the total load shedding, minimize interarea power exchange, or minimize the amount of active power adjustment. Then, control variables in these OPF models are switched shunt capacitor setting Q_C, reactive power injection from a static VAR compensator (SVC), DC tie line flow, phase shift transformer tap positions, and so on. Besides control variables and state variables, known parameters of the system are network topology, line parameters, voltage limits, transmission limits, generation limits, and generation cost coefficients. They are the parameters in the OPF problem.

The mathematical model (Equations 5.5 through 5.10) can be formulated with a general optimization formulation (Equations 5.11 through 5.14). Use **u** to represent vector of control variables and **x** to represent vector of state variables, then the OPF model can be formulated as

$$\text{Min } f(\mathbf{u}) \tag{5.11}$$

subject to:

$$\mathbf{g}(\mathbf{u},\mathbf{x}) = \mathbf{0} \tag{5.12}$$

$$\mathbf{h}(\mathbf{u},\mathbf{x}) \le \mathbf{0} \tag{5.13}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max} \tag{5.14}$$

$$\mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{x}_{\max} \tag{5.15}$$

where:

g is the equality constraints
h is the inequality constraints

5.3.2 Solution algorithms for optimal power flow

Interconnected power systems are usually large in size. A transmission system with different voltage levels covers a wide geographical area. For a power grid with an average size, for example, a power system that supplies the customers of a province or a state, there are hundreds of substations, thousands of transmission lines, and hundreds of generating units to supply customers in the area. Even on a simplified one-line diagram of a system of such a size, there are multihundred nodes and thousands of lines. It is obvious that the number of control variables and state variables is large. Power system is mainly an AC system. Voltages and currents are described as sinusoidal functions. Power system is well known for its nonlinearity in system control. Power flow equations, which are the network representation in OPF model, are nonlinear equations. Also, the number of nonlinear equations is large. If a system has 1,000 nodes, the number of nonlinear equations is at least 2,000. Classical OPF problem is a complicated nonlinear programming (NLP) problem. Besides nonlinearity, some variables in OPF problems are discrete variables, for example, positions of transformer tap changers, switch status of shunt capacitors, and reactors. This increases the difficulty of the problem and makes the problem a mixed-integer nonlinear programming (MINLP) problem. Moreover, OPF problem is a nonconvex problem, whose global optimal solution is not guaranteed.

Being a constrained optimization problem, OPF can be solved using constrained optimization algorithms, such as *dual methods*, *penalty methods*, and *augmented Lagrangian methods*. For solution algorithms and their applications to power system optimization problems, refer to Castillo et al. (2002). The classical method to solve OPF problem is gradient methods. The Lagrangian function is built considering both equality equations and inequality equations. The gradient of the Lagrangian function indicates the searching direction of each iteration until convergence is obtained. Kuhn–Tucker conditions are necessary conditions for optimization. Inequality constraints are difficult to be considered with gradient methods. To consider the inequality constraints of the problem, *complimentary slackness condition* is enforced for optimization. Karush–Kuhn–Tucker (KKT) conditions are necessary conditions for reaching an optimum for an inequality-constrained OPF problem.

Nonlinear OPF problem with both nonlinear objective function and nonlinear constraints can be solved using interior point method. Interior point method handles inequality constraints by adding slack variables to inequality constraints and converts all inequality constraints to equality constraints. Then log barrier terms are added to the objective function. The modified problem is solved with gradient method using Lagrangian function. Barrier parameters are forced toward zero during the iterations and close to zero when the iteration reaches a solution. Interior point method is a good solution method for solving nonlinear OPF problem, especially when full AC network model (power flow equations) are needed in the optimization.

Linear programming (LP) is a powerful optimization technique. LP can handle both equality constraints and inequality constraints, and it can guarantee an optimum solution for a linear optimization problem. LP algorithms and toolboxes are mature, and their optimum solutions are stable. To solve nonlinear OPF problem with LP solvers, we can linearize the nonlinear OPF problem. The nonlinear (quadratic or cubic) objective function can be linearized by using a linear cost function or piecewise linear cost functions. Power flow equations in the equality constraints of OPF problem are highly nonlinear. We can use DC power flow equations instead of AC power flow equations to linearize the equality constraints. The power transmission limits in inequality constraints can also be linearized, as DC power flow provides linear relationships between node power injections and line power flows. The convergence of linearized OPF problem is guaranteed; however, the solution might be a suboptimal solution of the original nonlinear problem, and the AC full network may not be fully respected. To improve the solution, we can use the solutions of linearized OPF to perform AC power flow and obtain the new operation points, then use the new operation points as initial points to solve the nonlinear problem, and iterate until better solution converges. Another disadvantage of linearize OPF problem is that reactive power is hard to be linearized. If reactive power needs to be studied in the OPF problem, it is not suggested to linearize the problem by replacing AC power flow with DC power flow. Nonlinear programming (NLP) solvers are well developed by mathematicians in recent decades. For example, the optimization solver MINOS is a well-known NLP solver. It is powerful enough to solve the nonlinear OPF problem with AC power flow constraints.

5.4 DC optimal power flow

DC optimal power flow (DCOPF) is a simplified version of AC OPF model by replacing AC power flow equations in the constraints with DC power flow equations. In power flow Equation 5.6, active power injection to node i in AC formulation is

$$P_{Gi} - P_{Di} = \sum_{k=1}^{N} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$$

Neglect self-connected branch (i, i), the power flow on branch (i, k) can be written as $P_{i,k}$.

$$P_{i,k} = |V_i|^2 G_{ik} - |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$$

If we use the assumption that voltages are close to per unit value 1, phase angle differences are small, and line resistance is much smaller than reactance, $|V_i| \approx |V|_k \approx 1$, $\sin\theta_{ik} \approx \theta_{ik}$, $\cos\theta_{ik} \approx 1$, and $r_{ik} \approx 0$, the formulation of $P_{i,k}$ can be simplified to the DC power flow as follows:

$$P_{i,k} = -B_{ik}(\theta_i - \theta_k) = (\theta_i - \theta_k)/X_{ik}$$

where X_{ik} is the reactance of branch (i, k).

Applying the simplified DC power flow equation to node active power injection functions, we have

$$P_{Gi} - P_{Di} = \sum_{\substack{k=1 \\ k \neq i}}^{N} P_{i,k} = \sum_{\substack{k=1 \\ k \neq i}}^{N} (\theta_i - \theta_k)/X_{ik} \qquad \forall i = 1, \ldots, N$$

This equation can replace Equation 5.6 in the OPF model to represent power balancing in the network.

The mathematical model of DC OPF can be represented as follows:

$$\text{Min } C_{\text{Total}} = \sum_{i=1}^{N} C_i(P_{Gi}) \tag{5.16}$$

subject to:

DC power flow constraints:

$$P_{Gi} - P_{Di} = \sum_{\substack{k=1 \\ k \neq i}}^{N} (\theta_i - \theta_k)/X_{ik} \quad \forall i = 1, \ldots, N \tag{5.17}$$

Generation limits:

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad \forall i = 1, \ldots, N \tag{5.18}$$

As all voltages are assumed to be 1, there is no reactive power or voltages in the DC OPF model. This model can be used for the problems in which only active power needs to be optimized, or in the cases where reactive power and voltage issues can be ignored. There is no power loss in the DC OPF model. As DC OPF is a LP problem, there is no convergence problem. The computation speed is much faster than solving an AC OPF model, particularly for a large system. The calculation error of DC OPF due to the approximations is around 3%–10% compared to the accurate results obtained from AC OPF model.

5.5 Security-constrained optimal power flow

OPF procures the optimal generation schedule by minimizing the total generation cost subject to power flow constraints and operation limits. The solutions are obtained based on normal state grid topology and load profiles. It has been recognized that the results of OPF obtained for the normal state may not be able to maintain the system safety operation after a line or a generator outage. Security issues need to be considered in the OPF model. The purpose is to obtain generation schedules that can keep the system in a normal state even after a major system disturbance. To implement this, the original OPF problem is solved subject to additional security-related constraints, which is *security-constrained OPF* (SCOPF).

Security constraints include the operation limits of post-contingency configurations for a given set of contingencies. In other words, the generation schedules obtained from solving SCOPF have rescheduling abilities to maintain the system in the normal state after contingencies. Contingency sets are provided by contingency analysis.

In OPF model (Equations 5.11 through 5.15), **u** and **x** are vectors of control variables and state variables, respectively,

$\mathbf{g}$ and $\mathbf{h}$ are equality constraints and inequality constraints, respectively. For a given set of contingencies, assume the number of system postcontingency configurations is c. For the kth contingency ($k = 1, \ldots, c$), control variables and state variables are represented by $\mathbf{u_k}$ and $\mathbf{x_k}$, respectively, for the postcontingency state. The control variables and state variables of the operation constraints for precontingency state (or base case) for SCOPF are $\mathbf{u_0}$ and $\mathbf{x_0}$. Operation equality constraints and inequality constraints for the base case are represented by $\mathbf{g_0}$ and $\mathbf{h_0}$. Equality constraints and inequality constraints for contingency cases are $\mathbf{g_k}$ and $\mathbf{h_k}$, respectively.

SCOPF problem formulations are generally formulated with two approaches: preventive SCOPF and corrective SCOPF. In preventive SCOPF model, the rescheduling of control variables (generation output schedule) is not allowed. In corrective SCOPF model, control variables (generation output schedule) could be different in contingency states than the base case. The mathematical models of the two approaches are presented as follows:

The preventive SCOPF model is formulated as

$$\text{Min } f(\mathbf{u_0}) \tag{5.19}$$

subject to:

$$\mathbf{g_0}(\mathbf{u_0}, \mathbf{x_0}) = \mathbf{0} \tag{5.20}$$

$$\mathbf{h_0}(\mathbf{u_0}, \mathbf{x_0}) \le \mathbf{0} \tag{5.21}$$

$$\mathbf{u}_{\min} \le \mathbf{u_0} \le \mathbf{u}_{\max} \tag{5.22}$$

$$\mathbf{x}_{\min} \le \mathbf{x_0} \le \mathbf{x}_{\max} \tag{5.23}$$

$$\mathbf{g_k}(\mathbf{u_0}, \mathbf{x_k}) = \mathbf{0} \qquad \forall k = 1, \ldots, c \tag{5.24}$$

$$\mathbf{h_k}(\mathbf{u_0}, \mathbf{x_k}) \le \mathbf{0} \qquad \forall k = 1, \ldots, c \tag{5.25}$$

$$\mathbf{x}_{\min} \le \mathbf{x_k} \le \mathbf{x}_{\max} \qquad \forall k = 1, \ldots, c \tag{5.26}$$

The corrective SCOPF model is formulated as

$$\text{Min } f(\mathbf{u_0}) \tag{5.27}$$

subject to:

$$\mathbf{g_0}(\mathbf{u_0}, \mathbf{x_0}) = \mathbf{0} \tag{5.28}$$

$$\mathbf{h_0}(\mathbf{u_0}, \mathbf{x_0}) \leq \mathbf{0} \tag{5.29}$$

$$\mathbf{u}_{\min} \leq \mathbf{u_0} \leq \mathbf{u}_{\max} \tag{5.30}$$

$$\mathbf{x}_{\min} \leq \mathbf{x_0} \leq \mathbf{x}_{\max} \tag{5.31}$$

$$\mathbf{g_k}(\mathbf{u_k}, \mathbf{x_k}) = \mathbf{0} \qquad \forall k = 1, \ldots, c \tag{5.32}$$

$$\mathbf{h_k}(\mathbf{u_k}, \mathbf{x_k}) \leq \mathbf{0} \qquad \forall k = 1, \ldots, c \tag{5.33}$$

$$\mathbf{u}_{\min} \leq \mathbf{u_k} \leq \mathbf{u}_{\max} \qquad \forall k = 1, \ldots, c \tag{5.34}$$

$$\mathbf{x}_{\min} \leq \mathbf{x_k} \leq \mathbf{x}_{\max} \qquad \forall k = 1, \ldots, c \tag{5.35}$$

$$\left|\mathbf{u_k} - \mathbf{u_0}\right| \leq \overline{\Delta\mathbf{u}} \qquad \forall k = 1, \ldots, c \tag{5.36}$$

Equations 5.20 through 5.23 and 5.27 through 5.31 are the same as those of the conventional OPF model. They represent the constraints for the precontingency state, or the base case. The optimal solutions are obtained for the control variables of precontingency state with a minimum value of the objective function, for example, the optimal generation schedules for a minimum total generation cost. Constraints in Equations 5.24 through 5.26 and constraints in Equations 5.32 through 5.36 impose postcontingency operation constraints (or security constraints) to the optimization problem to obtain the optimal precontingency generation schedules, which can be adjusted by corrective actions to still maintain the system security for any contingency state k in the contingency set. Constraint in Equation 5.36 applies to the allowed maximal adjustments for control variables to change from the precontingency state to a postcontingency state in corrective SCOPF. Preventive SCOPF can be regarded as a special case of corrective SCOPF with $\overline{\Delta\mathbf{u}} = 0 \;\; \forall k = 1, \ldots, c$.

SCOPF minimizes the objective function subject to power flow equations, operation limits and operation constraints of contingency states. The solution of SCOPF not only satisfies power flow equations and operation limits for postcontingency states but also ensures that the procured solutions of control variables $\mathbf{u_0}$, for example, generation schedules, are possible to move to a new steady state, $\mathbf{u_k}$ and $\mathbf{x_k}$, after a contingency k. In other words, solutions of SCOPF are feasible to be adjusted by corrective actions to satisfy operation and safety requirements of $N-1$ contingencies, or even $N-2$ contingencies. Corrective actions are usually predefined topology changes corresponding to the contingency. The system operator is used to defining a list of $N-1$ contingencies and $N-2$ contingencies at the system planning stage. For each contingency, a list of possible corrective actions is suggested that can be taken to maintain the system security after the occurrence of the contingency. SCOPF is a much more complicated optimization problem compared to standard OPF due to its large size of postcontingency constraints.

SCOPF problem is a NLP problem due to the nonlinearity of AC power grid. If discrete variables are formulated in the SCOPF problem, it becomes a MINLP. For a large power system, the size of SCOPF problem is large owning to the big number of nodes and contingencies considered. Solving the MINLP optimization problem for a large system while simultaneously solving all contingency constraints is not an easy job. It requires high-performance computers as well as feasible solution algorithms. It is necessary to simplify and reduce the size of the SCOPF problem in order to reach a feasible optimum solution in a reasonable time for practical applications. The problem can be simplified by reducing the number of considered contingencies in the contingency set or simplifying the postcontingency states.

In real systems, most contingencies may not affect optimal solutions; in other words, they are nonbinding constraints and do not really constrain the optimum. Only some contingencies constrain the optimum of the SCOPF problem. Including the full set of contingencies may result in extra numerical problems in the iteration process. It would be more efficient to find the smallest subset of contingencies that constrain the problem and result in the same objective value as the full set of contingency constraints. Only including the binding contingencies in the found subset to the SCOPF problem can reduce the size of the optimization problem. Linearization is another method to simplify the SCOPF problem. Especially for active power-related SCOPF problems, DC power flow equations are used. Guaranteed convergence to a global optimization solution is the major advantage of linearizing the SCOPF problem. However, the accuracy of the approximation in the linearization may be challenged, particularly for extremely nonlinear heavy-load conditions. For SCOPF problems that optimize reactive power, linearization approximation may have limitations in representing the original problem. The development of computation technologies and new optimization solution algorithms made it possible to apply SCOPF on large power systems for day-ahead generation scheduling or market operation. For real-time application of SCOPF, DC network model needs to be used to reach a feasible and acceptable solution within a short time. Parallel computation technologies and distributed computation are recently applied to solve the subset of postcontingency states independently.

5.6 Examples

5.6.1 Test system

In this section, we will use the IEEE standard system as examples to illustrate ACOPF, DCOPF, and SCOPF. IEEE standard systems include 14-bus system, 30-bus system, and 118-bus system. Here, we use IEEE 30-bus system for the examples. The topology structure of IEEE 30-bus system is shown in Figure 5.2.

The branch parameters of the system are the IEEE standard parameters that can be found online. There are 30 buses, 41 branches/lines, and 6 generators in the system. The 6 generators are located at buses 1, 2, 5, 8, 11, and 13. There are 22 buses that have electricity demand. The active power demand P_{Di} for bus i is listed in Table 5.1.

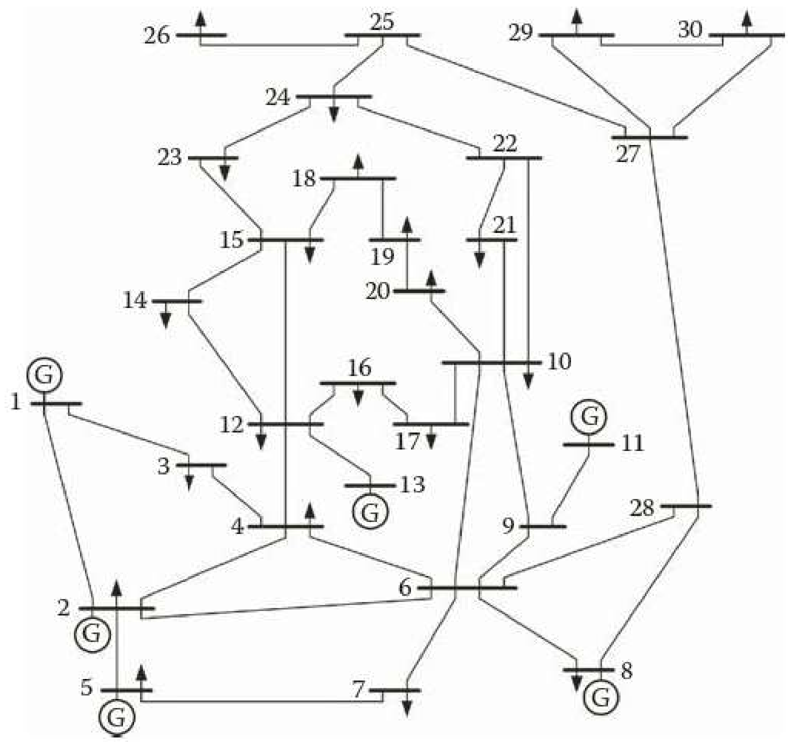

FIGURE 5.2 One-line diagram of IEEE 30-bus system.

Table 5.1 Active power demand for the 30-bus system (unit: MW)

i	P_{Di}	i	P_{Di}	i	P_{Di}	i	P_{Di}	i	P_{Di}	i	P_{Di}
1	0	6	0	11	0	16	3.5	21	17.5	26	3.5
2	21.7	7	22.8	12	11.2	17	9	22	0	27	0
3	2.4	8	30	13	0	18	3.2	23	3.2	28	0
4	7.6	9	0	14	6.2	19	9.5	24	8.7	29	2.4
5	94.2	10	5.8	15	8.2	20	2.2	25	0	30	10.6

Table 5.2 Coefficients of generation cost functions

Generator i	a_i (\$/MWh^2)	b_i (\$/MWh)	c_i (\$)	P_{Gi}^{Max} (MW)	P_{Gi}^{Min} (MW)
1	0.12	30	80	40	0
2	0.05	25	40	80	0
5	0.1	38	25	80	0
8	0.2	20	35	80	0
11	0.15	40	50	100	0
13	0.08	32	50	180	0

The generation cost function $C_i(P_{Gi})$ for each generator at bus i is assumed to be a quadratic function as follows:

$$C_i\left(P_{Gi}\right) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i$$

The coefficients a_i, b_i, and c_i for generators at bus i are listed in Table 5.2. Generation limits are also given in the table.

5.6.2 Example for AC optimal power flow

By solving the ACOPF model with full network constraints (Equations 5.5 through 5.10), the solution is generation schedules for the six generators. Using NLP optimization solver to solve the problem assuming transmission limits are not constrained, optimal generation schedule is obtained as given in Table 5.3.

From the results shown in Table 5.3, we can see that generators at bus 1 and bus 2 are relatively cheaper and scheduled to reach their generation limits 40 and 80 MW, respectively. The total load of the system is 283.4 MW. The optimal generation schedule in the table shows that the total generation for the optimal solution is 286.765 MW. The losses consumed by branch resistances are (286.765 − 283.4) = 3.365 MW. The standard 30-bus system has well-designed line parameters such that

Table 5.3 Optimal generation schedule for solving the ACOPF model

Generator i	Generation schedule P_{Gi} (MW)
1	40
2	80
5	32.267
8	57.887
11	11.819
13	64.792

the loss percentage is relatively low compared to a real power network. The optimized objective function, the total cost, is \$10,053.08.

5.6.3 Example for DC optimal power flow

If we only consider line reactance and solve the optimization model with DC load flow constraints by solving Equations 5.16 through 5.18, the generation schedule using DCOPF model is obtained. Table 5.4 shows the generation schedule results of DCOPF.

Same as solved by the ACOPF model, generators at bus 1 and bus 2 reach their generation upper limits. The total generation is 283.4 MW, which is the same as the total load. The reason is that transmission losses are not considered in DC load flow. A comparison of the results in Table 5.4 (DCOPF) with those in Table 5.3 (ACOPF) shows that the relatively expensive generator at bus 5 is dispatched to generate less because of the decrease in total generation. The total generation cost in this case is \$9,901.46.

Table 5.4 Optimal generation schedule for solving the DCOPF model

Generator i	Generation schedule P_{Gi} (MW)
1	40
2	80
5	25.63
8	57.815
11	10.42
13	69.536

5.6.4 Example for security-constrained optimal power flow

To obtain optimal generation schedules that satisfy the network operation requirements for both the normal state and contingency states, SCOPF model considering constraints for contingency states is proposed. In this example, we use AC load flow constraint-based SCOPF model for the case study. We first setup a contingency set, as shown in Figure 5.3.

We assume the contingency set includes 15 lines (i.e., other lines can also be in the contingency set). The lines in the contingency set are marked with a symbol "/" in Figure 5.3. Here, we assume the system satisfies the $N-1$ contingency criteria. It means, for any one of the 15 lines in the contingency set, if the line is out of service, the system is still safe and can be operated with the rest of the lines in the system. The total number of lines in the IEEE 30-bus system is 41; if any one line in the contingency set is disconnected, the system could operate with the rest 40 lines.

In this example, we use preventive SCOPF model (Equations 5.19 through 5.26) for the case study. Control variables in the model are generation outputs of the six generators, P_{Gi}, in normal state/base case. The constraints in Equations 5.20 through 5.23 are load flow constraints and inequality constraints, they

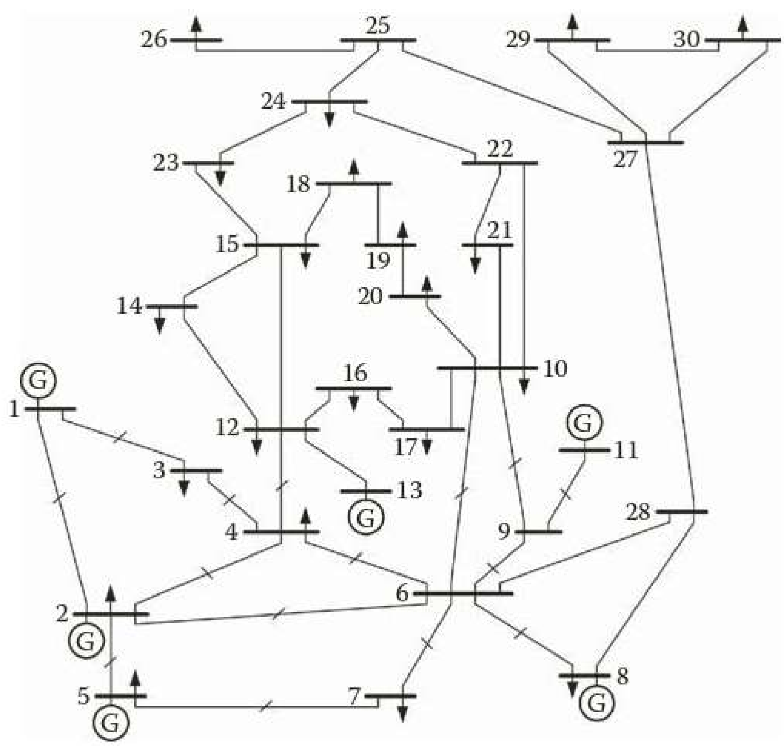

FIGURE 5.3 Contingency set of the IEEE 30-bus system for the example of SCOPF.

are same as in the ACOPF model. Constraints in Equations 5.24 through 5.26 are the load flow constraints and inequality constraints for all 15 contingency states. For each contingency, due to the loss of one line, the network matrix is different than base case.

AC power flow-based SCOPF is more complicated than normal ACOPF due to a large number of constraints. For this example, we can calculate the number of constraints for ACOPF and AC power flow-based SCOPF. In this example, the number of bus is 30, and the number of line is 41.

- For ACOPF, there are 30 active power balance equality constraints, 30 reactive power balance equality constraints, 6 inequality constraints for generator output limit, 30 * 2 inequality constraints for voltage upper and lower limits, 30 * 2 inequality constraints for phase angle upper and lower limits, and 41 * 2 inequality constraints for transmission line limits. In total, the constraint number is 268.
- For an AC power flow-based SCOPF, besides the 268 constraints for base case, for each line contingency, there are 266 constraints. The calculation for 266 is the same as earlier except there are 40 * 2 inequality constraints for transmission line limits due to the loss of one line in the contingency. As we have 15 contingencies in the contingency set in this example, the total number of constraints is 268 + 266 * 15 = 4258.

From the calculation, we can see that the calculation burden for a SCOPF model is much bigger than that for the regular ACOPF. In this example, we only put 15 lines in the contingency set. If we put all 41 lines in the contingency set, the number of constraints would be 268 + 266 * 41 = 11,174. In practical, a real network could have thousands of buses and thousands of lines. We can imagine how big a SCOPF problem will be. In real-world applications, to calculate SCOPF for a large system, usually DC load flow is applied for SCOPF. Otherwise, it is hard to obtain feasible solutions for such a large-size SCOPF problem if AC load flow equations, which are highly nonlinear, are included in the constraints.

In this example, to show the loss effect of the SCOPF model, we use AC load flow constraints for SCOPF. To reduce the calculation burden and obtain feasible solutions, we limit the number of contingencies in the contingency set to 15 instead of 41.

CHAPTER SIX

Unit commitment

6.1 Introduction

Electricity demand changes with human being's activities, as well as weather and season changes. There are more electricity consumptions during the daytime of weekdays, while electricity consumptions in the evening and weekends are relatively low. The electricity demand during the peak hours in a system could be more than double of the demand of the off-peak periods. The shape of a daily electricity load profile looks like a peak and a valley, as shown in Chapter 3. In different seasons, electricity consumptions change with temperatures, weather, and other factors. Load profiles of a system have their daily cycle, weekly cycle, seasonal cycle, and annual cycle. Within a load cycle, not all generating units need to be ON for the whole time period. Some of the generating units are only switched on during the peak hours and switched off when electricity demand is low. This is an optimization process that needs a mathematical model to optimize the schedule for switching on or off generating units. This problem is *unit commitment* (UC) problem in power system operations.

Unit commitment is a multiple time period optimization problem. The goal of optimization is to decide the ON/OFF schedules for all generating units in the system for a long time period, such as 1 week, 1 month, or 1 year. The optimization is based on generation costs, startup costs, the load profile, system operation constraints, and other limits. In this chapter, we will first use an illustrative example to explain the principles of Unit commitment. Then, the mathematic model of Unit commitment is presented and discussed.

6.2 Illustrative example of unit commitment

We first use an illustrative example to describe the basic concept of UC. Transmission network and losses are ignored in the example for simplicity. Assume there are three generators in the system. Their cost functions are same as used in Example 4.1, as shown in the following equations:

$$\begin{aligned}
&\text{Generator 1} \quad F(P_{G1}) = 0.148P_{G1}^3 + 199.4P_{G1} + 16425 \\
&\qquad\qquad 150\,\text{MW} \le P_{G1} \le 300\,\text{MW} \\
&\text{Generator 2} \quad F(P_{G2}) = 0.024P_{G2}^2 + 252.7P_{G2} + 16686 \\
&\qquad\qquad 300\,\text{MW} \le P_{G2} \le 600\,\text{MW} \\
&\text{Generator 3} \quad F(P_{G3}) = 0.109P_{G3}^2 + 168.0P_{G3} + 16639 \\
&\qquad\qquad 250\,\text{MW} \le P_{G3} \le 400\,\text{MW}
\end{aligned}$$

In this example, to simplify the calculation procedure, we select three generators from the four generators in Example 4.1. To supply the load in the system, there are seven options ($2^3 - 1$) to combine the three generators, which are shown in the following table. If the system has N generators, the options of committing generators is $2^N - 1$, while only the options that the total generation is higher that the load are selected as feasible solutions.

Options	1	2	3	4	5	6	7
Generator 1	ON	–	–	ON	ON	–	ON
Generator 2	–	ON	–	ON	–	ON	ON
Generator 3	–	–	ON	–	ON	ON	ON
Maximum output (MW)	300	600	400	900	700	1,000	1,300

For a load of 500 MW, options 1 and 3 do not have enough capacity to supply the load. The maximum outputs of generator 1 and generator 3 are lower than the load, 500 MW. Options 2, 4–7 are feasible solutions to supply the load. The next question is which option is the best solution. As the generators that are ON in each option is given, we can use economic dispatch (ED) or optimal power flow (OPF) to obtain the least-cost generation schedule for each option. In this illustrative example, transmission network is ignored, so we use the equal-λ method introduced in Chapter 4 to obtain ED result for each option.

Option 2: The solution is: generator 2 is ON and generates 500 MW (ignore losses). The coal consumption of this solution is

$$F(P_{G2})\big|_{P_{G2}=500} = 0.024P_{G2}^2 + 252.7P_{G2} + 16686 = 149{,}039 \text{ kg}$$

Option 4: Both generator 1 and generator 2 are ON. The optimal generation outputs are obtained for two generators using equal-λ method. The procedure is shown as follows:

$$\frac{\mathrm{d}F(P_{G1})}{\mathrm{d}P_{G1}} = \frac{\mathrm{d}F(P_{G2})}{\mathrm{d}P_{G2}} = \lambda \quad \Rightarrow \quad \begin{cases} 0.296P_{G1} + 199.4 = \lambda \\ 0.048P_{G2} + 252.7 = \lambda \end{cases}, \text{ and}$$

$$P_{G1} + P_{G2} = 500 \text{ MW}$$

The solutions are $P_{G1} = 224.7$ MW, $P_{G2} = 275.3$ MW, and $\lambda = 265.93$ kg/MWh. The coal consumption is

$$F(P_{G1})\big|_{P_{G1}=224.7} + F(P_{G2})\big|_{P_{G2}=275.3} = 156{,}776 \text{ kg}$$

Option 5: Both generator 1 and generator 3 are ON. The optimal generation outputs are obtained for two generators using equal-λ method. The procedure is shown as follows:

$$\frac{\mathrm{d}F(P_{G1})}{\mathrm{d}P_{G1}} = \frac{\mathrm{d}F(P_{G3})}{\mathrm{d}P_{G3}} = \lambda \quad \Rightarrow \quad \begin{cases} 0.296P_{G1} + 199.4 = \lambda \\ 0.218P_{G3} + 168 = \lambda \end{cases}, \text{ and}$$

$$P_{G1} + P_{G3} = 500 \text{ MW}$$

The solutions are $P_{G1} = 151$ MW, $P_{G3} = 349$ MW, and $\lambda = 244.09$ kg/MWh. The coal consumption is

$$F(P_{G1})\big|_{P_{G1}=151} + F(P_{G3})\big|_{P_{G3}=349} = 138{,}456 \text{ kg}$$

Option 6: Both generator 2 and generator 3 are ON. The optimal generation outputs are obtained for two generators using equal-λ method. The procedure is shown as follows:

$$\frac{\mathrm{d}F(P_{G2})}{\mathrm{d}P_{G2}} = \frac{\mathrm{d}F(P_{G3})}{\mathrm{d}P_{G3}} = \lambda \quad \Rightarrow \quad \begin{cases} 0.048P_{G2} + 252.7 = \lambda \\ 0.218P_{G3} + 168 = \lambda \end{cases}, \text{ and}$$

$$P_{G2} + P_{G3} = 500 \text{ MW}$$

The solutions of aforementioned three equations are $P_{G2}=91.2\,\text{MW}$, $P_{G3}=408.8\,\text{MW}$, and $\lambda=257.12\,\text{kg/MWh}$. However, the maximum output of generator 3 is $P_{G3}^{\text{Max}}=400\,\text{MW}$. So, the optimal solution considering output limits is $P_{G2}=100\,\text{MW}$, $P_{G3}=400\,\text{MW}$. The coal consumption is

$$F(P_{G2})\big|_{P_{G2}=100}+F(P_{G3})\big|_{P_{G3}=400}=143{,}476\text{ kg}$$

Option 7: All three generators are ON. The optimal generation outputs are obtained for them using equal-λ method. The procedure is shown as follows:

$$\frac{\mathrm{d}F(P_{G1})}{\mathrm{d}P_{G1}}=\frac{\mathrm{d}F(P_{G2})}{\mathrm{d}P_{G2}}=\frac{\mathrm{d}F(P_{G3})}{\mathrm{d}P_{G3}}=\lambda$$

$$\Rightarrow\begin{cases}0.296P_{G1}+199.4=\lambda\\0.048P_{G2}+252.7=\lambda,\ \text{and}\\0.218P_{G3}+168=\lambda\end{cases}$$

$$P_{G1}+P_{G2}+P_{G3}=500\text{ MW}$$

The solutions obtained from previous four equations are $P_{G1}=172\text{ MW}$, $P_{G2}=-50\,\text{MW}$, $P_{G3}=378\text{ MW}$, and $\lambda=250.35\text{ kg/MWh}$. As generation output cannot be negative, we set $P_{G2}=0\,\text{MW}$. Then, only generators 1 and 3 are ON, which becomes option 5.

The feasible options and their solutions for load 500 MW are summarized and listed in the following table.

$P_{\text{Load}}=500$ MW				
Option	2	4	5	6
Generator 1 (MW)	–	224.7	151	–
Generator 2 (MW)	500	275.3	–	100
Generator 3 (MW)	–	–	349	400
Total coal consumption (kg)	149,036	156,776	$\boxed{138{,}456}$	143,476

It is obvious that option 5 is the best solution. Generators 1 and 3 are committed for the hour with load 500 MW, while the output of generator 1 is 151 MW and that of generator 3 is 349 MW. The coal consumption is 138,456 kg, which is marked with a square.

To supply the system load, there are different options of combining different generators. For each set of generator combination, the optimal generation schedule is obtained using ED problem. It shows that the optimal solution are different if different generators are committed for the hour. ED problem is a subproblem of UC problem. UC problem searches for the best unit combination for an hour, whereas ED solves for optimal generation schedule for the unit combination.

If the system load varies from 400 to 1,100 MW in 1 day, we can obtain the best UC option and generation schedules for each load level following aforementioned procedures. To implement this manually, we repeat earlier calculations and obtain the results for load levels of 400, 500, 600, 700, 800, 900, 1,000, and 1,100 MW. Here, a step of 100 MW is applied. If needed, a smaller step can be used for a more detailed solution. Feasible solutions for different load levels are obtained and shown in the following tables. For each load level, the UC option with the lowest coal consumption is marked with a square.

$P_{\text{Load}} = 400$ MW

Option	2	③	4	5	6
Generator 1 (MW)	–	–	211	108.5	–
Generator 2 (MW)	400	–	189	–	9.3
Generator 3 (MW)	–	400	–	291.5	390.7
Total coal consumption (kg)	121,606	[101,279]	130,391	114,675	117,953

$P_{\text{Load}} = 500$ MW

Option	2	4	⑤	6
Generator 1 (MW)	–	224.7	151	–
Generator 2 (MW)	500	275.3	–	100
Generator 3 (MW)	–	–	349	400
Total coal consumption (kg)	149,036	156,776	[138,456]	143,476

$P_{\text{Load}} = 600$ MW

Option	2	4	⑤	6	7
Generator 1 (MW)	–	238.7	200	–	183.8
Generator 2 (MW)	600	361.3	–	200	22.5
Generator 3 (MW)	–	–	400	400	393.7
Total coal consumption (kg)	176,946	183,573	[163,504]	169,465	180,134

$P_{Load} = 700$ MW				
Option	4	⑤	6	7
Generator 1 (MW)	252.6	300	–	197
Generator 2 (MW)	447.4	–	300	103
Generator 3 (MW)	–	400	400	400
Total coal consumption (kg)	210,785	190,844	195,935	205,698

$P_{Load} = 800$ MW			
Option	4	⑥	7
Generator 1 (MW)	266.6	–	211
Generator 2 (MW)	533.4	400	189
Generator 3 (MW)	–	400	400
Total coal consumption (kg)	238,409	222,885	231,670

$P_{Load} = 900$ MW			
Option	4	⑥	7
Generator 1 (MW)	300	–	224.7
Generator 2 (MW)	600	500	275.3
Generator 3 (MW)	–	400	400
Total coal consumption (kg)	266,511	250,315	258,055

$P_{Load} = 1{,}000$ MW		
Option	⑥	7
Generator 1 (MW)	–	238.7
Generator 2 (MW)	600	361.3
Generator 3 (MW)	400	400
Total coal consumption (kg)	278,225	284,853

$P_{Load} = 1{,}100$ MW	
Option	⑦
Generator 1 (MW)	252.6
Generator 2 (MW)	447.4
Generator 3 (MW)	400
Total coal consumption (kg)	312,064

Table 6.1 Unit commitment and generation schedule results for different load levels

Load (MW)	400	500	600	700	800	900	1,000	1,100
Generator 1	–	151	200	300	–	–	–	252.6
Generator 2	–	–	–	–	400	500	600	447.4
Generator 3	400	349	400	400	400	400	400	400

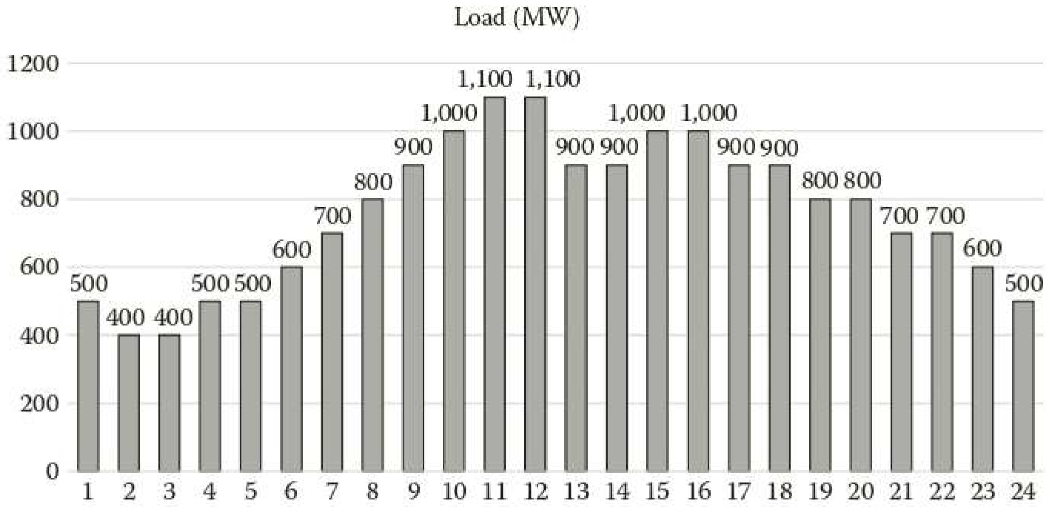

FIGURE 6.1 A 24-hour load profile.

According to the results obtained in the tables, the best (optimal) UC options are summarized for different load levels and listed in Table 6.1.

From the table, a priority list is obtained for generators with the increase in load. The priority list is (1) generator 3; (2) generators 1 and 3; (3) generators 2 and 3; (4) generators 1, 2, and 3.

Load in 1 day varies for different hours. A 24-hour load profile is provided in Figure 6.1. There are two peak load periods, one is in the morning before noon time and the other one is in the afternoon. The load in the night is low.

Considering both the UC results in Table 6.1 and the 24-hour load profile in Figure 6.1, we can obtain the UC schedule for 24 hours as shown in Table 6.2.

The table shows that generator 3 is always ON for the whole day. Generator 1 is switched on at 4 a.m. in the morning when the load increases. With further increase in load, generator 1 reaches its limit and swaps with generator 2, generator 2 is ON. When the load reaches the peak value, 1,100 MW, all three

Table 6.2 Unit commitment results for 24 hours

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Load	500	400	400	500	500	600	700	800	900	1,000	1,100	1,100
Generator 1	ON	–	–	ON	ON	ON	ON	–	–	–	ON	ON
Generator 2	–	–	–	–	–	–	–	ON	ON	ON	ON	ON
Generator 3	ON	ON	ON	ON	ON	ON	ON	ON	ON	ON	ON	ON
Hour	13	14	15	16	17	18	19	20	21	22	23	24
Load	900	900	1,000	1,000	900	900	800	800	700	700	600	500
Generator 1	–	–	–	–	–	–	–	–	ON	ON	ON	ON
Generator 2	ON	ON	ON	ON	ON	ON	ON	ON	–	–	–	–
Generator 3	ON	ON	ON	ON	ON	ON	ON	ON	ON	ON	ON	ON

generators are ON to supply the load. After 12 p.m., generator 1 is OFF, both generators 2 and 3 are remained ON to supply the load until 8 p.m. in the evening. From 9 p.m., generator 2 is shut down and generator 1 is ON to supply the load with generator 3 until 1 a.m. Then, generator 1 is OFF, only generator 3 is kept running to supply the load during off-peak period until 4 a.m.

We use a three-generator system to show the concept of UC. The method first enumerates all unit combinations for each load level and runs ED problem to obtain the generation schedule for each unit combination, then the most economic unit combination is determined for each load level. A priority list is obtained for the least-cost UC. If the load profile is available, the system operator can decide the sequence of switching ON/OFF the units according to the priority list. The method can be used to manually obtain UC solutions, except that the calculation burden would be huge if the number of generators in the system N is large. There will be $2^N - 1$ options for N generators, and ED needs to be calculated for all feasible options of all load levels. Here, we use this enumeration method to do UC, as a good example to explain the concept of UC, as well as the relationship between UC problem and ED problem. However, the method considers only generation costs/fuel consumptions, and the balance of generation and load. System operation limits and generating unit constraints are not considered in the method. Using optimization and computation technologies, it is possible to include all operation constraints in an optimization problem to mathematically formulate UC problem. In the next section, the mathematical model is presented to formulate all constraints and considerations of UC problem.

6.3 Mathematical model of unit commitment problem

In power system operations, generation schedules are made for each hour. For example, the generation schedule obtained from ED or OPF is the hourly generation output (in MW or kW) for each generator. The energy accumulated in the hour is the product of generation output and 1 hour, which is in MWh or kWh. Similarly, UC problem schedules unit ON/OFF for each hour. Electricity load has daily cycle, weekly cycle, seasonal cycle, and annual cycle. UC is a multiple time period optimization problem to decide the unit operation schedules from 24 hours

(1 day) to 8,760 hours (1 year). The optimization problem formulated for UC includes the ED subproblem within hourly optimization. The links between time periods are represented by some interhour constraints.

The objective function of UC problem is to minimize the total cost, including operation costs, startup costs, and so on, within the whole period, say 8,760 hours for 1 year. The objective is obtained subject to various constraints, including system operation constraints, generating unit constraints, intertime period constraints, constraints to start up/shut down units, and so on.

6.3.1 Variables of unit commitment problem

6.3.1.1 Control variables For the system operator, the purpose of running UC program is to obtain the unit ON/OFF schedules and generation schedules for each hour. Control variables in the problem are ON/OFF states of generation unit i during time period t and generation output of unit i during time period t. If the total number of generators in the system is N and the total number of time period is T, then $i = 1,\ldots,N$, and $t = 1,\ldots,T$. In many cases, the UC problem is simulated for 1 year (8,760 hours), so it is common to see the expression that $t = 1,\ldots,8760$.

In this chapter, the control variable, generation output of unit i at time period t, is represented by the symbol $P_{G,i}^t$. The upper and lower limits for unit i are $P_{G,i}^{\text{Max}}$ and $P_{G,i}^{\text{Min}}$, respectively. If reactive power is considered, reactive power of unit i at time period t is represented by $Q_{G,i}^t$ and its upper and lower limits are $Q_{G,i}^{\text{Max}}$ and $Q_{G,i}^{\text{Min}}$, respectively. For PV nodes, their voltages, V_i^t, are control variables.

We use symbol W_i^t to represent the state of unit i during time period t. Variable W_i^t is a binary variable, which represents the ON or OFF state of a unit at time t.

$$W_i^t = 1, \quad \text{unit } i \text{ is ON at time } t$$

$$W_i^t = 0, \quad \text{unit } i \text{ is OFF at time } t$$

6.3.1.2 State variables Voltages of non-PV nodes V_i^t and phase angles θ_i^t are state variables in the UC problem. State variables change with the changes in control variables that adjust power generations and switch ON/OFF generation units.

6.3.2 Objective function

The objective function of UC problem is similar as OPF. It minimizes the total cost of all scheduled generations in all time period, plus the startup cost for switching ON generating units during the whole time period. The formulation of objective function is

$$\text{Min}\left(\sum_{t=1}^{T}\sum_{i=1}^{N} C_i(P_{G,i}^t) + \text{Startup Cost}\right) \tag{6.1}$$

where $C_i(\bullet)$ is the cost function of generator i, which is similar to the generation cost function introduced in ED and OPF problems. The selection of unit i during time period t is determined by binary variables W_i^t as shown in constraints (Equations 6.2 and 6.3).

As UC problem schedules ON/OFF schedules for generation units. The cost of starting a unit should be included in the objective function. The startup cost is mainly caused by thermal units. As the temperature and pressure of the furnace of a thermal unit moves slowly, certain amount of energy is needed to bring the unit online. This results in startup cost. The amount of startup cost needed depends on the statues of the unit. If the unit is started from a cold machine, more startup energy is needed. If the unit is just turned OFF and its furnace temperature is not too low compared to the operation temperature, less startup energy is needed. For different types of generation units, their startup costs are different and can be formulated with linear cost functions or nonlinear cost functions, which also rely on whether they were treated as *cooling* machines or *banking* machines when turned down.

6.3.3 Unit constraints

6.3.3.1 Generator output limits Each generation unit has its maximum generation capacity $P_{G,i}^{\text{Max}}$. The output of a unit cannot be higher than $P_{G,i}^{\text{Max}}$ at any time period. The generation output upper limit is represented by

$$P_{G,i}^t \le W_i^t \times P_{G,i}^{Max} \qquad \forall i = 1,\ldots,N \text{ and } \forall t = 1,\ldots,T \tag{6.2}$$

Steam turbine-based generators have minimum load limitations. For example, some generators are required to operate at least 30% of their generation capabilities. The minimum generation requirements are set due to fuel combustion stability requirements and generation design requirements. For other types of generators, the minimum loading requirements are not so strict. The generation output lower limit for UC problem is expressed as

$$W_i^t \times P_{G,i}^{Min} \le P_{G,i}^t \qquad \forall i = 1,\ldots,N \text{ and } \forall t = 1,\ldots,T \tag{6.3}$$

Different from the generation output limits in the OPF problem, $P_{G,i}^{\mathrm{Max}}$ and $P_{G,i}^{\mathrm{Min}}$ in Equations 6.2 and 6.3 are multiplied by a binary control variable W_i^t, which determines whether unit i is selected. If $W_i^t = 0$, $P_{G,i}^t$ is limited to be zero, which means unit i is not selected for time t. If $W_i^t = 1$, $P_{G,i}^t$ is limited as $P_{G,i}^{\mathrm{Min}} \le P_{G,i}^t \le P_{G,i}^{\mathrm{Max}}$, which is the same as in ED and OPF problems.

In most of the conventional generators, such as coal-fired power plants, oil-fired power plants, and gas turbines, the outputs can be controlled to follow generation schedules within the limits. Their outputs are controllable. In hydro power stations with water reservoirs, the power outputs are also controlled by controlling the amount of water flowing through water turbines. The generation outputs of these controllable generators are control variables in the optimization problem for UC.

Besides controllable generators, there are some other generators in power systems, their outputs are not controllable. For example, power outputs of renewable energy generations are intermittent and not controllable. Power outputs of wind turbines and solar panels rely on the weather and the availability of wind and sunshine. Power outputs of run-of-river hydro power stations depend on water flow in the river. Without energy storage, renewable energy generation outputs are variable, and they are uncontrollable power sources. However, similar to variable loads, renewable energy generations can be forecasted according to historical data and weather forecast. In OPF and UC problems, loads in power balance equations are given with forecasted loads. Similarly, renewable energy generation outputs could be treated as given when doing the optimization, if accurate forecast of wind and solar are provided. Some researchers use different types of stochastic OPF methods to consider renewable energy generation fluctuations, which is also a way to solve the problem. Different from renewable energy generations, nuclear power lacks fluctuations. Power outputs of nuclear power stations remain relatively constant due to operation requirements. In a simple way, power outputs of these uncontrollable generation resources can be treated as given in the UC optimization problem, especially if the purpose of UC is to determine only unit schedules, instead of detailed real-time optimal generation schedules.

6.3.3.2 Generation unit ramping constraints Fossil fuel-fired power plants convert thermal energy to mechanical energy and then to electrical energy. The moment of inertia determines the time-delay characteristics of the energy conversion system. The electrical output of a unit cannot change by more than a certain amount over a period of time. Generation schedules between two adjacent time periods are constrained by unit ramping abilities. The ramping ability is defined as the maximum amount of power that a generator can increase or decrease within a certain time period, say, 1 hour or 15 minutes. For an hourly operation, ramp up rate is the maximum amount of power a unit can increase within 1 hour, represented by $\Delta P_{G,i}^{\text{up,max}}$. Ramp down rate is the maximum amount of power that a unit can reduce within 1 hour, represented by $\Delta P_{G,i}^{\text{down,max}}$.

Maximum ramp-up constraints

$$P_{G,i}^{t+1} - P_{G,i}^{t} \le \Delta P_{G,i}^{\text{up, max}} \tag{6.4}$$

Maximum ramp-down constraints

$$P_{G,i}^{t} - P_{G,i}^{t+1} \le \Delta P_{G,i}^{\text{down,max}} \tag{6.5}$$

6.3.3.3 Generation unit operation time constraints Thermal generation units have minimum up-time and down-time constraints. This is because the furnace temperature of a thermal unit changes gradually. It takes some hours to bring a unit online. Due to such temperature constraints, a thermal UC or decommitment is restricted by minimum up-time constraints and minimum down-time constraints.

Minimum up-time constraints

Once a thermal unit is ON, it should run at least for a minimum up-time, $t_i^{\text{up,min}}$. The number of hours the unit is already on, t_i^{up}, should be less than $t_i^{\text{up,min}}$.

If $W_i^t = 1$ and $t_i^{\text{up}} < t_i^{\text{up,min}}$, then $W_i^{t+1} = 1$

Minimum down-time constraints

Once a thermal unit is OFF, it cannot start immediately and needs to be shut down for at least a time of $t_i^{\text{down,min}}$. The number of hours the unit is already OFF, t_i^{down}, should be less than $t_i^{\text{down,min}}$.

If $W_i^t = 0$ and $t_i^{\text{down}} < t_i^{\text{down,min}}$, then $W_i^{t+1} = 0$

6.3.3.4 Other unit constraints To dispatch generation units in a power system, some other practical operation requirements need to be considered for UC, such as crew constraints, must-run constraints, fuel constraints, and so on.

6.3.3.4.1 Crew constraints Turning a thermal unit ON or OFF requires the crew from the power plant to operate the unit. If there are more than one units in a power plant, they should not be committed to be ON or OFF at the same time, since there may not be enough crew to take care of the starting up procedures of multiple units. Having at most one unit of a power plant to be ON or OFF during a time period is one of the unit constraints in the UC problem.

6.3.3.4.2 Must-run constraints Some generators, due to their strategic locations in the electric network, have to be ON during certain time periods or certain seasons to maintain system stability or support voltages. These generators are must-run units. The commitment schedules for must-run units are fixed in the UC problem. In other words, the binary variables for a must-run unit i are preset as 1 during the must-on time periods t, $W_i^t = 1$. Must-run units are usually determined according to operation experiences and stability simulations.

6.3.3.4.3 Fuel constraints Generation companies purchase fuels from primary energy market. Energy prices fluctuate in international energy markets. Most generation companies have their strategies to hedge risks in energy market. The storage of fuels (coal, oil, gas, etc.) in a power plant may be limited by fuel prices and their purchase agreements in the fuel market. The available fuels during a given time period should be considered as constraints of UC problem.

6.3.4 System constraints

6.3.4.1 Power balance constraints Similar to ED and OPF problems, UC problem is optimized subject to power balance constraints. Power flow Equation 6.6 provides power balance constraints with complete full network models.

$$
\begin{aligned}
P_{G,i}^t - P_{D,i}^t &= \sum_{k=1}^{N} |V_i^t||V_k^t|\left(G_{ik}\cos\theta_{ik}^t + B_{ik}\sin\theta_{ik}^t\right) \\
&\qquad \forall i = 1,\ldots,N \quad \forall t = 1,\ldots,T \\
Q_{G,i}^t - Q_{D,i}^t &= \sum_{k=1}^{N} |V_i^t||V_k^t|\left(G_{ik}\sin\theta_{ik}^t - B_{ik}\cos\theta_{ik}^t\right) \\
&\qquad \forall i = 1,\ldots,N \quad \forall t = 1,\ldots,T
\end{aligned}
\tag{6.6}
$$

However, in traditional UC problem, it may not be necessary to use AC power flow equations as power balance constraints. The reason is that UC problem is already a complicated multiperiod mixed-integer nonlinear programming (MINLP) problem, the nonlinear AC power flow equations ($2 \times N \times T$ equations) would make the UC problem even more complicated and hard to obtain feasible solutions. On the other hand, since UC aims to obtain unit ON/OFF schedules only, detailed generation schedules are procured by ED or OPF. It is good enough if simplified power balance equations are used. DC load flow Equation 6.7 are usually used in UC problem for power balance constraints.

$$P_{G,i}^t - P_{D,i}^t = \sum_{\substack{k=1 \\ k \neq i}}^{N} \frac{\left(\theta_i^t - \theta_k^t\right)}{X_{ik}} \qquad \forall i = 1,\cdots,N \quad \forall t = 1,\cdots,T \quad (6.7)$$

Sometimes, we even ignore transmission network and use one simplified power balance Equation 6.8. However, in this case, transmission network constraints cannot be considered.

$$\sum_{i=1}^{N} P_{G,i}^t = P_{\text{Load}}^t \quad \forall t = 1,\cdots,T \qquad (6.8)$$

where P_{Load}^t is the total system load during time period t.

6.3.4.2 Network constraints Energy transmitted in a transmission line is limited by the line transmission capability. The outputs from some generators may be limited if their outputs result in transmission line overloading, or transmission congestions. Transmission line constraints (Equations 6.9 and 6.10) should be considered in the UC problem.

$$P_{i,k}^t = \left|V_i^t\right|^2 G_{ik} - \left|V_i^t\right|\left|V_k^t\right|\left(G_{ik}\cos\theta_{ik}^t + B_{ik}\sin\theta_{ik}^t\right) \qquad (6.9)$$

$$P_{i,k}^t \le P_{i,k}^{\text{Max}} \qquad (6.10)$$

where $P_{i,k}^t$ is the power flow on line (i, k) during time period t and $P_{i,k}^{\text{Max}}$ is the power transmission limit on line (i, k).

If DC load flow is used, simplified transmission constraints (Equations 6.11 and 6.12) are used.

$$P_{i,k}^t = \frac{\left(\theta_i^t - \theta_k^t\right)}{X_{ik}} \qquad (6.11)$$

$$P_{i,k}^t \le P_{i,k}^{\text{Max}} \qquad (6.12)$$

6.3.4.3 System reserve constraints To maintain system reliability, the total amount of generation available at any time period should be higher than the load. In contingency events, if generators or lines are lost, the system should have enough reserve capacities to make it up. System reserves have spinning reserves, backup reserves, and supplement reserves. Spinning reserves are the generation capacities synchronized to the system. It can increase power generations in a short time (e.g., a few minutes) once the reserve is called for a contingency state. Compared to spinning reserves, backup reserves take a little bit longer to increase their generation outputs to fulfill the requirement. Supplement reserves are the generation capacities not synchronized to the system. Once it is called for in a contingency state, it takes some time (e.g., 30 minutes) for the generator to start and synchronize to the system.

The solutions of a UC problem should guarantee there are enough spinning reserves in the system during any time period. The available spinning reserve in a system is calculated as total generation capability minus the total generation output of the hour. The total generation capability is the sum of the generation capacities of all running generators in the system, which is $\sum_{i=1}^{N} P_{G,i}^{\text{Max}}$. The available spinning reserve should be higher than the system reserve requirement R^{req} for any time t, as shown in Equation 6.13.

$$\sum_{i=1}^{N} P_{G,i}^{\text{Max}} - \sum_{i=1}^{N} P_{G,i}^{t} > R^{\text{req}} \qquad \forall t = 1, \ldots, T \tag{6.13}$$

If we use DC load flow, which ignores losses, DC power balance Equation 6.8 is applied. Then, the reserve constraint can be expressed as Equation 6.14.

$$\sum_{i=1}^{N} P_{G,i}^{\text{Max}} - P_{\text{Load}}^{t} > R^{\text{req}} \qquad \forall t = 1, \ldots, T \tag{6.14}$$

The next question is how to decide the reserve requirement R^{req} of a system. The purpose of having reserves is to guarantee that there are enough reserve capacities even the largest generating unit or interconnection in the system is lost. A deterministic criterion, which sets the capacity of the largest generation unit as system reserve requirement, is commonly used by power companies. The other commonly used deterministic criterion is to set a certain percentage (say, 15% or 20%) of the system peak load as the system reserve requirement. Besides the two deterministic criteria, some power companies use probabilistic criteria to decide the system reserve. The probabilistic criteria are to take into account the capacities of all committed units as well as their outage rates.

6.4 Solution algorithms for unit commitment problem

UC problem is a complicated MINLP problem. The number of variables is large due to big number of nodes and multiple time periods. If the number of generators in the system is N, for each hour, the total combination for N generators is $2^N - 1$. For T time period, the possible combinations could reach $(2^N - 1)^T$. Enumeration method is not a practical idea for a large system to solve the UC problem, although it could reach accurate solutions. Commonly used solution algorithms for UC problem include priority-list method, dynamic programming method, and Lagrange relaxation method.

Priority-list method is similar to the illustrative example presented in Section 6.2. The principle is to create a unit priority list for each load level. For a complicated system, the priority list could be obtained with a simple way by using full-load average generation cost instead of calculating ED problem. This is based on the assumptions that generation cost functions are linear between zero load and full load, and startup costs are fixed. Priority-list method is simple and easy to implement without heavy computation burdens. The optimality/accuracy of the method is acceptable for making preliminary UC schedules. Detailed OPF solutions could be obtained afterward for each hour.

Dynamic programming method is commonly used for solving MINLP problems. It is implemented in multiple steps. For each step, possible solutions are enumerated and ranked in a priority order. Paths with higher priorities are selected and then move to next step, and do the same for the next step. In this way, the dimensionality of the problem is reduced and maintained to be a solvable optimization problem. The final solutions may not be the global optimal solution. However, it is acceptable for solving a complicated optimization problem like

UC problem. Dynamic programming algorithm is commonly used by MINLP solvers in commercial optimization software.

Lagrange relaxation solution algorithm is also popular in solving MINLP-based UC problems. Similar to the ones used for OPF, the Lagrange functions are obtained and solved. The solution methods for using Lagrange relaxation in MINLP problem can be found in books about Optimization.

6.5 Summary

Unit commitment problem is a multiple time period optimization problem that optimizes the total operation cost and all startup costs within a long time period, say 1 year, by making generation unit commitment schedules. The minimization of operation cost and startup cost must be done for the whole time period to obtain generation schedules. Some of the constraints link time periods together. The constraints are unit ramping constraints, startup state constraints, running time constraints, and so on. UC problem is a more general problem compared to ED problem, and ED problem is a subproblem of UC problem. ED problem optimizes only for 1 hour with given running units.

CHAPTER SEVEN

Electricity market overview

7.1 Traditional power industry

The first power generation and transmission system in the world that became operational at Pearl Street in New York in 1882 was a DC system with DC generators and DC lines. AC generation technologies were developed around the same period. The first three-phase AC transmission system was put into operation in 1893. To have lower power losses and larger transmission capabilities, higher voltage levels are preferred for long distance power transmission. Compared to DC systems, AC systems have the inherent feature of changing voltage levels with the help of AC transformers. In the past century, long distance AC transmission systems have been constructed all over the world for almost all power systems.

Traditional power industry is a monopoly system. An integrated power company owns and runs power plants, transmission lines, and distribution lines, as well as provides customer services to end users. The system operator of the power company is responsible for monitoring and controlling all the facilities in the system to maintain system security and reliability. Besides maintaining real-time power balancing, the system operator runs economic dispatch to minimize the total generation cost. Economic operation is one of the functions run by the system operator of a power grid.

To improve the reliability and efficiency of system operation, neighboring power grids are interconnected for power exchange and emergency support. Large-scale interconnected power systems are very complicated in operation and control. A fault event could result in a cascading blackout for a wide area. For example, the Northeast blackout of 1965 in the United States was initially caused by an incorrect tripping of a line, which was due to the incorrect setting of a protective relay. The loads were transferred to other lines and caused cascading line outages. The blackout affected over 30 million people, and an area of 207,000 km^2 had no electricity supply for 13 hours. The *federal commission* was appointed to follow up the blackout. It was recommended that a real-time monitoring and control system is necessary for interconnected power systems. Supervisory control and data acquisition (SCADA) system and energy management system (EMS) began to be developed since then. SCADA/EMS is a computer system that uses real-time measurement data to monitor, control, and manage a power system. SCADA/EMS developed by different vendors have been used in the control centers all over the world. SCADA system measures voltage, current, and other information of the power grid and transmits the information to the computer servers located at the control center. The information is used by the EMS applications for network monitoring, generation scheduling, system control, and security analysis. SCADA/EMS plays a very important role in system security and economic operations for almost all power systems in the world. The development of EMS began in 1960s and 1970s, when power systems everywhere in the world were vertically integrated monopoly systems. EMS was designed with centralized control aspects for the centralized power system operation and control for that period.

In a traditional power system, through SCADA/EMS in the control center, the system operator makes generation schedules for all generating units based on their generation costs; runs automatic generation control and automatic voltage control; maintains generation reserves for real-time operation; and implements contingency analysis, dynamic security analysis, and fault analysis for security operations. Both generating units and transmission grids are operated and monitored by the system operator, who is responsible for the security and reliable operation of the whole integrated system.

7.2 Deregulation of power industry

In 1990s, some countries started the deregulation process for power industry following the deregulation of other monopoly industries, such as railways, airlines, and gas pipelines, and so on. Since AC power system was adopted in 1890s to support long distance power transmission, the monopoly business model has been applied to power industry for around one century. One of the incentives for changing the industry is introducing competitions to power generations aiming for a higher energy efficiency. The restructuring of power industry is to deregulate the regulated power industry and unbundle the integrated power system into separated sections: generation, transmission, distribution, and customer services, and so on.

7.2.1 Unbundling of integrated power systems

Traditionally, integrated power company generates electricity from their own generators, transmits, and distributes energy to customers while providing customer services. Power system expansion planning, system operation and control, asset management, transmission services, and customer services are all supported and provided by the integrated power company. To unbundle the integrated system, generation, transmission, and distribution assets are separated and owned by independent companies, such as generation company (GenCo), transmission company (TransCo), and distribution company (DisCo). Besides the unbundling of power system assets, the functions, such as system operations and customer services, originally conducted by the integrated company are separated and carried on by newly generated entities. New entities in a deregulated system include independent system operator (ISO)/transmission system operator (TSO), power exchange, energy trader/marketer, electricity retailer, and so on. The services originally provided with energy by the integrated company are also separated, and some of them need to be purchased from service providers. The unbundled services include ancillary services, customer services, and so on (Figure 7.1).

In the deregulated power system, ISO/TSO conducts the tasks originally undertaken by the system operator of the integrated power company. However, the ISO/TSO is not supposed to know the generation cost of each generator for system dispatch, while the system operator of an integrated power system dispatches generators according to their generation costs. This is because ISO/TSO is independent of generation companies. Generation cost information is commercially

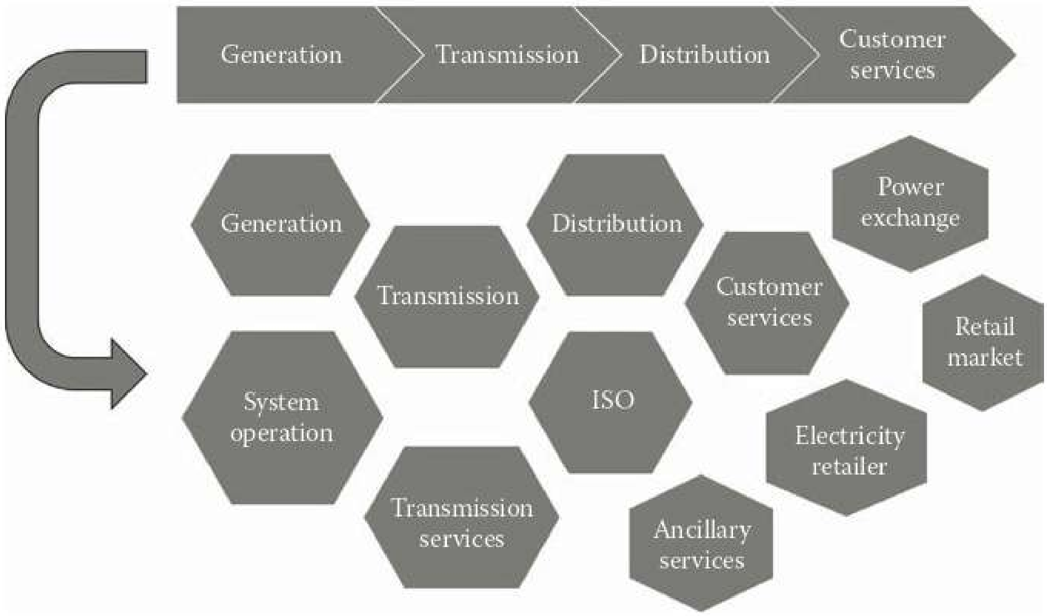

FIGURE 7.1 Unbundling of integrated power systems.

confidential information, which is not disclosed to the ISO/TSO. Power control center is the brain and the soul of a power system for its security and reliable operation, irrespective of whether the system is a deregulated system or a vertically integrated system.

Energy Exchange (or Power Exchange) provides the market platform for trading electricity and electricity-related products for market participants. The central clearing house of energy exchange provides clearing and settlement services for power-related transactions occurring in the energy exchange. This is new in a deregulated power system. In a vertically integrated power system, energy is generated and delivered to all end-users who are connected to the network of the power company. The delivered energy is settled between the power company and the customer with regulated electricity tariff. There is no trading or market for electricity in the integrated system. Economic issues and overall operation costs are considered by the system operator by economic operation. In most deregulated power systems, power exchange center is established for power trading and settlement of power transactions. Power exchange center and power control center are parallel centers responsible for market operation and system operation, respectively.

Energy traders start to join electricity markets when generation assets, transmission and distribution networks are unbundled. They trade electricity and electricity-related products in the market via energy exchange platform or other market mechanisms. Traders make financial gains from price differences of electricity products. Both energy traders and energy exchange are new in the power section and appear only after power industry restructuring.

Electricity retailer is another new participant to restructured power systems. To obtain power supply, every electricity customer must be connected to one power distribution network. In the vertically integrated system, the owner of the distribution network, which is utility or Distribution Company, is responsible for supply of electricity to all customers connected and gets paid with the regulated flat rate. In the deregulated system, customers are free to enter the market to look for preferred power supplier based on electricity prices and customer services offered by the suppliers, and other factors. The number of customers in a system is huge. The market for small customers is electricity retail market. Electricity retailers purchase electricity from the electricity wholesale market as large customers, and then sell electricity in electricity retailer markets to small customers, with additional services in some cases. Energy companies and grid owners could act as electricity retailers. Moreover, distribution networks should be open to retailers and customers. Open access of distribution network makes it possible for retailers to enter the market as well as for customers to change power suppliers.

Ancillary services are those that are provided by the system operator to maintain the system security and reliability operation. Ancillary services include frequency regulation, operation reserves, reactive power support/voltage control, black-start, and so on. In the traditionally integrated power system, these services are provided together with energy without any additional charges. The system operator dispatches generators to provide energy as well as the services to maintain real-time power balancing and system security. In a deregulated system, as generation companies are independent of the system operator and transmission network, generation companies will count the costs of providing these services and charge for providing ancillary services. So, ancillary services are separated from the integrated system as a new type of chargeable services to be procured by the system operator from ancillary service providers.

Customer services in the traditional system are provided with electric power as part of the services accompanying the power supply. After unbundling of power systems, customer services are separated from the integrated system. Power distribution companies, power suppliers, and retailers provide the services to customers in different ways.

7.2.2 Restructured power system for market operation

The structures and the operation modes of power systems have been restructured since power industry deregulation. The structure change is to include market operation in the daily system operation. The structures before and after deregulation are shown in Figure 7.2.

The figure shows that the structure of a traditional system is a transmission and distribution network that connects generators and electricity customers. The structure of the system supports power delivery from generation side to demand side. In a restructured system, physical connections of transmission and distribution lines are same as in the traditional system. However, the ownerships are separated into transmission companies and distribution companies. Independent generation companies own generation assets. Electricity is traded in the electricity wholesale market and electricity retail markets. Besides generators and customers, electricity retailers emerge as new participants to the power system and electricity market. Retailers play an important role in introducing electricity market to the distribution network and customer sides.

There is usually one electricity wholesale market within a regional transmission grid. The ownerships of distribution networks could be all-sorts of companies, or owned by utilities. There could be multiple retail markets in an electricity distribution area. Generation companies sell most of their electricity in the wholesale market. Large customers and retailers purchase electricity from the wholesale market. Retailers then go

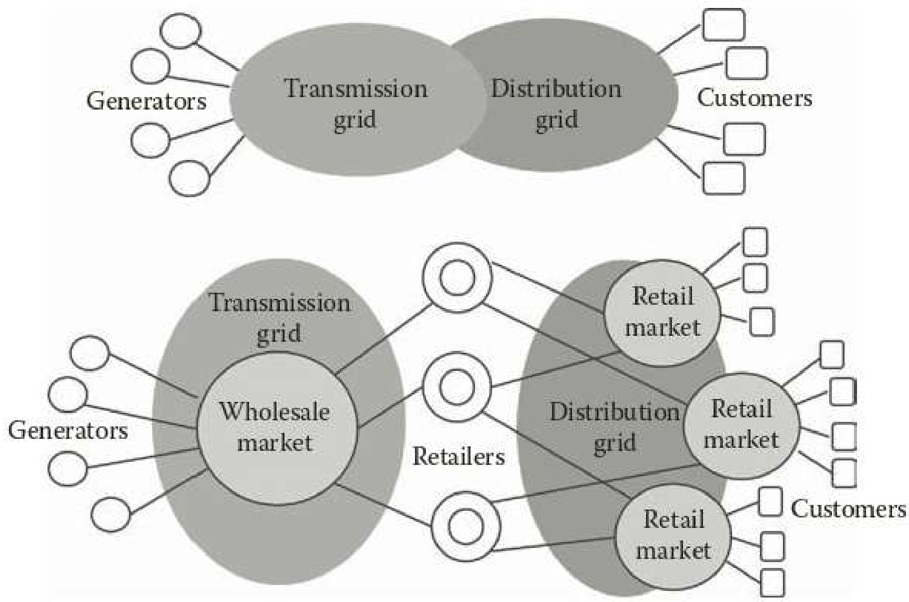

FIGURE 7.2 Power system structure change for market operation.

to retail markets to sell electricity to small customers within the distribution area. Distributed generators (DGs) can participate in the retail market to supply electricity to customers in the same distribution grid.

The system structure becomes more complicated in the restructured system with more market entities and market participants. The introduction of markets into power systems bring more flexibilities as well as competitions to both electricity supply and electricity consumption.

7.3 Power deregulation in different countries

Chili (and then Argentina) has been regarded as the first country that introduced market mechanisms in the traditional system in 1978, although the power industry in Chili now has been changed back to the vertically integrated structure. The incentives for introducing market concepts in the South American system were mainly due to the weather of late 1970s, which were El Nino and La Nina years. The La Nina resulted in less rain in South America. Lack of water in reservoirs caused power generation shortages in South American countries where hydro power dominates the generation capacities. This was the reason that power trading and market concept were introduced to the power systems of Chili and Argentina, and later to other South American countries, although the success of the markets was limited in the end.

Most deregulated power systems started deregulation procedures from 1990s. The years of starting power industry deregulations in different regions are shown in Figure 7.3.

FIGURE 7.3 Power industry deregulation in the world.

The leading incentives of power system restructuring vary in different countries/systems. Also, the diversities in restructuring processes of different countries are obvious.

Among currently restructured power systems, Great Britain was the pioneer country that started a power market in England and Wales in late 1980s, although the original market model was replaced by a new model in 2002. Following Great Britain, Australia started power system restructuring in early 1990s and New Zealand in 1994. Norway established Nord Pool in 1991 for power exchange among Nordic countries. The structure of Nordic electricity market is stable and successful these years. Members of European Union (EU) began the process of power deregulation in late 1990s and early 2000s under EU-level market rules. EU countries restructured their own power systems and trade power across borders with other EU countries in the Energy Exchanges established for European markets. In the United States, California started power deregulation in 1992. Then, New York, New England, PJM (Pennsylvania–New Jersey–Maryland) in the northeast of the United States, and Texas started power system deregulation around 1998. Other states in Midwest and Southwest followed power deregulations in early 2000s. California modified its market structure in 2002 after the Energy Crisis of 2001 in California. China restructured power industry in 2002 by separating power generations from the traditionally integrated system. Five power generation companies own the majority of generation assets in the country. Two transmission companies, State Power Grid and China Southern Power Grid, own and control the national-wide power grid. In 2016, China started electricity retail markets by introducing competitions to power distribution and power supply to end users.

7.3.1 Great Britain

The restructuring of power sector in Great Britain started with power system privatization in 1980s. Generation assets and generation companies were separated from the transmission company, the National Grid Company (NGC), which is also the TSO of the market. Distribution assets were privatized into regional electricity companies that own distribution networks. Regional electricity companies are responsible for supplying electric energy to customers and selling electricity as electricity retailers. The first electricity market commenced in 1990 in England and Wales with the introduction of Power Pool for electricity wholesale market. Generators offer electricity generation quantities and prices to Power Pool. Large customers and electricity retailers bid for electricity from Power Pool.

Generation offers are ranked in the ascending order as a supply curve. Customer bids are ranked in the descending order as a demand curve. Generators with lower offers have higher priority to sell electricity, while customers with higher bids have higher priority to buy electricity. The Power Pool matches generators and customers according to the priority lists of offers and bids. The cross point of generation supply curve and demand curve determines the market clearing price. Generators offered lower prices and customers bid for higher prices are selected in the market. The concept of price priority list and the mechanism of market clearing price developed with the Power Pool have been widely adopted in the later electricity market designs and applied for market clearing for different systems. We can say that the Power Pool model is a pioneer market model that provides a theoretical basis for later mechanism designs of electricity markets in the world. Power Pool-based market has been operational for over 10 years. In 2001, it was reviewed and replaced by a new market arrangement. The new market structure is known as New Electricity Trading Arrangement (NETA).

NETA has an entirely different trading structure compared to the pool-based market. The design of NETA adopts a new market framework to accommodate a large number of bilateral contracts and energy trading in power exchange. Bilateral contracts dominate the major portion of power transactions, which is an effective market design in coordination with power system reliable and security operations. In 2005, the market area of NETA was extended from England and Wales to a bigger region that includes Scotland. The market is upgraded to British Electricity Trading and Transmission Arrangements, which is called BETTA.

In recent years, due to climate change and global warming issues, more renewable energy generations have been installed in the system. British electricity market was reformed in 2010 to support renewable energy targets. New components in the market include fixed prices for low carbon generations, carbon prices, capacity market, and emission performance related mechanisms. The electricity retail market was reformed in 2012. The retail market introduced customer flexibilities to the system as well as competitions to power distribution and energy supply.

7.3.2 Australia and New Zealand

The restructuring of power system in Australia commenced in early 1990s, following the power sector privatization in Great Britain. The major restructuring was done to establish the National Electricity Market (NEM) in southern and eastern Australia. The territory of NEM includes five states, New South Wales, South Australia, Queensland, Victoria, and Tasmania, and the Australian Capital area. The market operator is Australia Energy Market Operator (AEMO). AEMO is responsible for both power system operation and market operation of NEM. The National Electricity Market includes a power pool and a spot market. Generators, electricity retailers, and large customers trade electricity in the wholesale market through power pool. Retailers sell electricity to customers in electricity retail markets. AEMO is a *doing everything* operator on the NEM territory. It is responsible for generation dispatch, grid operation, system reliable and safety operation, market operation, and market settlement in the NEM region.

New Zealand reformed the power industry during the similar time as Australia. The wholesale electricity market, New Zealand Electricity Market (NZEM), was established in 1996. It has the world first nodal pricing-based market model. Transmission grid is owned and operated by the state-owned transmission company, Transpower. M-co, which is the former Electricity Market Company, manages the electricity market as a market regulator. Generators and retailers trade in the power pool and spot market, and electricity is traded and transmitted across the nation-wide transmission grid. Electricity price is determined for each node on a half-hour basis.

7.3.3 Nordic countries

The Nordic market was commenced in 1991 in Norway. One of the incentives to introducing power exchange in Norway was the lack of water in the hydro power reservoirs in 1991, which was a dry year. In Norway, 99% of electricity is generated by hydro power. Nord Pool was established in Oslo of Norway as a Power Exchange. Sweden, Finland, West Denmark, and East Demark joined the Nordic electricity market operated by Nord Pool. Denmark has two separate transmission systems in West Denmark and East Denmark, respectively. Power system restructuring in Nordic countries is similar. Generation companies are separated from national power grids. Generators and retailers/large customers trade electricity in the wholesale market. Electricity retail markets are open to industrial customers, commercial customers, and residential customers. Transmission grids are owned and operated by state transmission companies (TransCos). In Norway, Statnett is the TSO, which owns and operates the Norwegian transmission grid. Svenska Kraftnat (Svk) is the TSO of the Swedish transmission grid. In Finland, Fingrid is the TSO. The Danish transmission system is owned and operated by Energynet.dk.

In Nordic electricity market, electricity is traded physically through bilateral contracts and in the electricity spot market. Zonal price model is adopted for the market. Price zones are separated subject to transmission congestions. The electricity sport market is operated on an hourly basis. The products traded in Nord Pool electricity financial market include Futures, Forward, and Options. Spot market and financial market are settled in the Nord Pool Settlement Center.

Generators and interruptible loads provide real-time electricity balancing services in the ancillary service market. Balancing resources provide quantity-price pairs for upregulation and downregulation on a 15-minute basis in the balancing power market. The TSO of each country maintains power balance in their system and runs the electricity balancing market for the country.

7.3.4 United States

Power system restructuring in the United States began in 1992 after the launch of the Energy Policy Act of 1992. The Order 888 issued by the Federal Energy Regulatory Commission (FERC) in 1996 indicates the open access of transmission grid. California started the first electricity market in the United States and established power exchange (PX) and California ISO (CAISO) for the electricity market in the state. Some other states followed California and started power system deregulation in late 1990s. In the electricity markets of the United States, the ISO, which is a nonprofit entity, operates the deregulated power system and runs the electricity market. The ISO does not own any generation or transmission assets. It is independent from generation companies and grid companies. The ISO is responsible for maintaining the system reliability for one or more than one control areas. Besides California ISO, other ISOs in the United States are New York ISO (NYISO), ISO New England (ISO-NE), PJM (Pennsylvania—New Jersey—Maryland), Midwest ISO (MISO), Electric Reliability Council of Texas (ERCOT), and Southwest Power Pool (SPP). The territories controlled by the ISOs can be seen from their names.

At the beginning of market model design in the mid-1990s, there were debates about zonal pricing, power pool model, and bilateral contract model. Following their operation background, PJM and ISO-NE used uniform price-based zonal pricing mechanism around 1998. Some issues, such as market power subject to network constraints, were observed. At the beginning of 2000s, California electricity wholesale market experienced extremely high price spikes during some critical peak time periods while electricity retail prices were regulated. Oil, gas, and electricity market prices were so high to sustain during the year. California energy crisis in 2002 demonstrated that electricity interruptions and power market designs could have big impacts on economy. It is necessary to include power system security analysis in the electricity market model and to consider the possibility of physical power transfer in the network.

In 2002, standard market design (SMD) was developed and applied in the electricity markets of American Northeast, including the markets operated by PJM, NYISO, and ISO-NE. The standard market design is based on the locational marginal pricing mechanism, which is a nodal pricing approach. The energy market is cleared with traditional security-constrained economic dispatch (SCED) model, or security-constrained optimal power flow (SCOPF) model. SCED and SCOPF models used in the market optimize the benefits of power supply according to generation offers and customer bids. Energy traded through bilateral contracts are formulated as generation constraints in the optimization model of SCED and SCOPF. As security-constrained optimization model is the clearing approach of the standard market design, security constraints considered traditionally by the system operators are not missing in the market design. Both market operation and security-based system operation are satisfied in the model.

However, due to transmission network constraints and different auction prices, transmission congestions may occur. Congestions may result in very high price differences between nodes because of the nodal pricing mechanism. The financial instrument, financial transmission right, is developed to hedge the risks of transmission congestions. Real-time power balancing is obtained in the ancillary service market. In the standard market design of the United States, frequency regulation and reserves are obtained in the ancillary service market. Different from European market models, frequency regulation services and reserve services are cleared simultaneously with energy. In other words, the procurement of ancillary services is co-optimized with energy market in the SCED model or SCOPF model.

The systems adopted the standard market design have proved to be run stable and successful. ERCOT and MISO applied similar market design concepts in their markets. Later, California modified its market design and replaced the previous power exchange-based market with a new market structure. The revised version of California market has similar market concept as other electricity markets in the United States.

7.3.5 Europe

Traditionally, integrated power systems were liberalized in the countries of European Union (EU) after three consecutive EU legislative packages addressing different market aspects were issued between 1996 and 2009. With the completion of EU's internal energy market (IEM), energy companies and electricity suppliers can enter the energy market of other EU member countries. Electricity customers are free to switch to other energy suppliers. The goal of EU's internal energy market is to have an open access network and an open market that power can flow freely across country borders without any technical or regulatory barriers.

The market design of IEM has similar architecture as Nordic electricity market. TSOs control the network in their countries/regions. The European Network of Transmission System Operators for Electricity (ENTSO-E) was founded in 2009. ENTSO-E helps in setting market-related rules and regulations and the collaboration of European TSOs. Similar to Nordic market, most energy is traded through bilateral contracts in forward market and over-the-counter (OTC) market. The rest is traded in day-ahead and hourly-ahead spot markets through several European Power Exchanges. Procuring balancing services are the responsibilities of TSOs.

7.3.6 China

The power industry restructure of China started in 2002. Five generation companies were separated from the former state power company. Two grid companies, State Power Grid (SG) and China Southern Power Grid (CSG) were established. The five generation companies own most generation capacities in China. CSG controls the high-voltage power grid that interconnects five provinces in the South. SG controls the rest, which is the major portion of high-voltage power grid in China. After the deregulation in generation side, the market reformation in China was suspended in around 2004. One of the reasons was electricity shortages in some provinces due to fast economy growth for around one decade. Installed generation capacity has been more than doubled from 2004 to 2014. After years of suspension of electricity market, Chinese government opened electricity retail market in 2016. Many electricity retail companies are established. Retail licenses are issued to electricity retailers to sell electricity to industrial customers and residential customers. At the current stage, retailers purchase energy through long-term contracts or in a pool-like market on a monthly basis. The government and power exchange center are working on setting up an electricity market model that is suitable to Chinese power systems.

7.4 Electricity retail markets

In the deregulation process of almost all countries, the first step is to introduce competitions to the generation side and establish electricity wholesale market. Transmission network is separated and owned by the transmission company, and the system is operated by an ISO. Introducing competition at the retail level is an extension of the competition at the wholesale level. The next step of power system restructuring is to open the electricity retail market.

Traditionally, a customer pays electricity bill to the distribution company or utility who owns the distribution network that the customer is connected to. Customer has no choice of electric power suppliers. In a wholesale market, electric power suppliers/electricity retailers sign bilateral contracts with generators and bid in the spot market for purchasing electricity. Providing customers with options for choosing electric power suppliers is regarded as extending the competition from wholesale level to retail level. A system becomes a more competitive market by introducing both wholesale competition and retail competition.

New Zealand is the first country that started electricity retail market in 1994. Nordic countries opened electricity markets to customers just after their power system deregulation in 1991. By 2000, only Sweden, Norway, Finland, Germany, and Great British in Europe had their electricity retail markets opened to customers. By 2007, almost all European countries had established electricity retail markets following their electricity wholesale markets. In the United States, currently, 14 states and Washington DC have retail options to customers. The states are mainly within the control territories of ISO New England, PJM, and MISO, plus Texas State controlled by ERCOT. Some other states have limited retail choices. In the states with retail choices and retail markets, more than half of industrial and commercial loads have experiences of switching to competitive power suppliers. This is because the benefits of switching power suppliers are large considering the large amount of electricity consumed by industrial and commercial customers, while the percentage of residential customers that switch suppliers is relatively low. Whether the residential customers have retail choices show the level of competition in a retail market. Texas has a competitive retail market for residential customers. Around 70% of residential customers switched power suppliers. An online system for switching suppliers and choosing electricity plan makes it much easier for residential customers participating in the electricity retail market.

In an electric power system with retail options, the number of retailers varies from time to time. Once the market is opened to retailers and customers, the original distribution company or the utility in the region automatically becomes a retailer that already has customers connected to their distribution networks. Because of the open access to the grid, new retailers enter the retail market and offer competitive prices to customers. Some customers, especially those having big electricity bills, such as industrial customers and commercial customers, are attracted by the low prices offered by new retailers and are willing to switch their power suppliers. The feasibility of switching power suppliers and how convenient it is to switch usually affect the customer switching rate of a retail market. With new retailers entering the market, the market has become competitive. There are usually three types of retailers: (1) original distribution company/utility in the region; (2) retail companies founded by generation companies, energy companies or other energy-related enterprises; (3) new electricity retailers without generation or distribution assets. According to past experiences and statistics, the number of electricity retailers is usually bigger at the beginning when a retail market starts. After operation for years, the market becomes more competitive, and the number of retailers becomes stable. The number of retailers reduces especially when the market is open to residential customers, which means the market is more competitive.

Electricity retail market is hard to be a perfectly competitive market due to the limitation of network connection. For a given area, not all retailers can provide power supply services to customers in the area, only retailers who are authorized or licensed can do it. Previous distribution companies and generation companies have preponderance over other types of retailers. Electricity retail market is a relatively concentrated market. In a country or a state, only a few retailers have more than 5% market shares. A retailer owns more than 5% market shares (or customers) of a country is called a *major retailer*. In relatively competitive markets, such as Sweden, Finland, Denmark, and Germany, the number of major retailers is around three to four. Moreover all major retailers in total hold around 45%–60% market share of the country. In relatively centralized retail markets, such as Spain and Belgium, three to four major retailers hold around 90% market share of the country; and the largest retailer has a dominant position and owns more than 50% market share. In some systems, like Italy and France, only one major retailer dominates around 90% of the market share, and no any other retailer has a market share of more than 5%.

7.5 Overview of electricity market operation

7.5.1 Markets

Commodities traded in the electricity market are usually active power due to the tradition that only active power is charged in the electricity bill. Energy consumption by a customer is measured as the amount of active power used within 1 hour. So, the fundamental commodity traded in the market is energy (in kWh). Actually, both active power and reactive power flow in the AC power system. Reactive power is not charged in the traditional electricity bills. Besides reactive power, frequency control and reserves are provided to maintain system security operations. These are provided by the system operator of traditionally integrated power company as part of transmission services. However, generations are unbundled and the system operator is independent in the deregulated system, these services, which are called ancillary services, have to be procured from generators to maintain system security operations. In an electricity market, the fundamental commodity is energy, which is traded in the energy market. Ancillary services, which are needed for system operation, are procured in the ancillary service market. Similar to other commodity markets, electricity price risk is hedged in the electricity financial market. Electric energy is the major commodity in the energy market. Ancillary services are procured and financially compensated to maintain the real-time power balancing and system security for energy market. Electricity financial tools hedge the price risk in the energy market.

In this section, the overview of energy market, ancillary service market, and electricity financial market are presented. Details about market operations will be illustrated in Chapter 8.

7.5.1.1 Energy market Energy is traded in the energy market with different timescales. In a traditional power system, the generation schedule is composed of long-term seasonal schedule, weekly schedule, and day-ahead and hourly-ahead schedule. The purpose is to follow the changes of load patterns and short-term load fluctuations. Similar to traditional generation scheduling, energy is traded in a long-term through bilateral contracts, and in a short-term through electricity spot markets. Bilateral contracts can be signed between generation companies and electricity retailers/large customers in advance. Electricity spot market is a place for market participants to trade energy in a short-term, usually day-ahead and hourly-ahead of real-time power delivery. They are called day-ahead spot market and hourly-ahead spot market, respectively.

Energy market is the major market in a deregulated power system. Customers purchase only active power from the energy market. For electricity users, electricity is priced on the basis of the amount of active power consumed in 1 hour. Electricity pricing is an important issue for an energy market, especially for the design of a successful energy market. Well-designed electricity pricing mechanisms can provide a fair platform for energy trading, while mitigating the exercising of market power. Electricity pricing mechanisms could be different for different systems. However, marginal price is the principle of electricity pricing. It is applied in various energy market designs with different forms.

7.5.1.2 Ancillary service market Ancillary services are those services procured by the system operator to maintain system reliable and safety operations. Before power system deregulation, power companies generated and delivered energy to customers, and at the same time provided transmission services to guarantee the energy delivery. After deregulation, generation companies were independent from the grid company and the system operator. Some services that were supplied by generators had to be procured by the system operator in the ancillary service market. Ancillary services include frequency regulation, system reserve, voltage control and reactive power support, black start, and so on. Generation companies usually provide ancillary services. However, a variety of ancillary service providers have appeared in recent years. For example, owners of energy storage systems can provide frequency regulation services; controllable loads and demand response can be considered equivalent to a generation source to provide frequency regulation service or reserve service.

Participants of ancillary service markets are suppliers that own generation resources or demand resources. Currently, frequency regulation and reserve are the two major ancillary services procured in the market. Market mechanisms for ancillary services differ in different systems. In the United States, reserves and regulations are settled with energy transactions simultaneously. In most European systems, energy market is settled first and frequency regulation is then settled in the balancing market after electricity spot prices are obtained in the energy market.

7.5.1.3 Electricity financial market Similar to other commodity markets, there are price risks in the electricity market. The risks come from uncertainties of demand, transmission congestions, and fluctuations of primary energy prices, and so on. The volatility of electricity prices could be much bigger than that of energy market and stock market. Financial tools have been developed to hedge price risks in most mature electricity markets. Futures, forward, and options are the major energy derivatives applied in current electricity financial markets. They are usually traded in power exchanges, or traded in OTC markets. In the United States, the financial tool, financial transmission right (FTR), is developed to hedge the risk of high prices caused by transmission congestions.

7.5.2 Market participants

Unlike integrated power systems, an unbundled power system has various market participants. The participants include Generation Company, Transmission Company, Distribution Company, ISO, Electricity Retailer, Load Serving Entities, Load Aggregator, Marketer, Broker, Power Exchange, large customers, and so on.

Generation company (GenCo), transmission company (TransCo), and distribution company (DisCo) own power stations and power grids. They run the system for power generation and delivery. Generation companies provide electric power and ancillary services. Transmission companies provide power transmission services. Transmission tariffs in many countries are regulated by the regulator. In some systems, part of transmission costs is covered by transmission congestion charges. Distribution companies provide power distribution services to end users, in most systems, with regulated distribution tariffs. Transmission and distribution grids are supposed to be open access. The ISO monitors and controls the power system, implements power transactions in day-ahead and hourly-ahead generation schedules, but procures ancillary services to maintain system reliability and security. Power Exchange is responsible for energy transactions, including day-ahead spot market, hourly-ahead spot market, bilateral transactions, OTC transactions, electricity futures, forward, and options.

Generation companies are the sellers in the electricity wholesale market. Electricity retailers, load serving entities, load aggregators, marketers, and so on are the buyers in the electricity wholesale market. Although they have different names, they all perform as buyers in the electricity market and purchase energy directly from generation companies or from electricity spot markets. Some electricity retailers or load serving entities also own and run distribution networks. They provide energy distribution services to customers in their networks, as well as perform as retailers in the electricity retail markets. Electricity retailers are energy suppliers and sellers in the electricity retail market. End users, including large customers and residential customers, are buyers in the electricity retail market.

7.5.3 Market operations and its timeline

The timeline of electricity market operation is similar as the timeline for traditional generation scheduling, which is composed by long-term schedule, short-term generation scheduling, and real-time operation. In electricity markets, most transactions are signed through long-term bilateral contracts, electricity futures contracts, and forward contracts. In short-term, day-ahead and hourly-ahead spot markets are complements to long-term transactions. Real-time power balancing is maintained in electricity balancing market or frequency regulation market. The timeline of electricity market transactions is shown in Figure 7.4.

Long-term bilateral contracts signed between sellers and buyers could be several years before the date of energy transaction. OTC markets open to market participants in some countries. An energy trade can be executed between two market participants in an OTC market. Bilateral contracts and transactions in the OTC market have the similar effect as having long-term generation schedules in the traditional power system operation. A big portion of bilateral contracts can ensure a stable operation of electricity market and reduce the chances of price spikes. In most mature electricity markets, it shows that a high percentage of energy is traded through long-term bilateral contracts. Besides bilateral contracts and OTC markets, electricity derivatives are traded in the Power Exchange to hedge the market risk. Derivatives commonly traded in the electricity

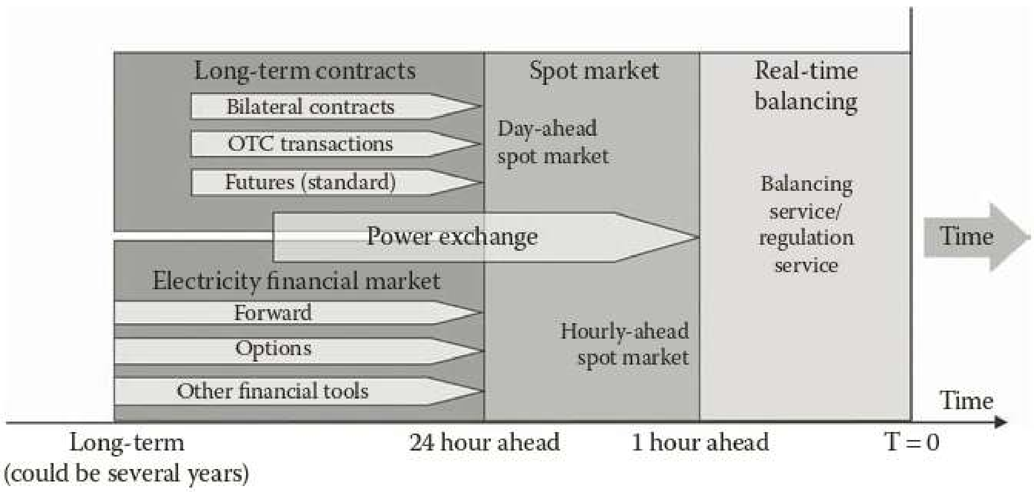

FIGURE 7.4 Timeline of power transactions.

financial market are futures with standard contracts, electricity forward, and options. In some countries, there are other financial tools applied in the electricity financial market. Different from bilateral contracts, electricity futures and forward can be traded for multiple times before the day of real-time energy delivery. Electricity derivatives can be used by market participants to hedge risk of spot price. Arbitrager will use these financial tools to exploit benefits.

Electricity demand in a system changes frequently. It is almost not possible to have a 100% accurate load forecast. Electricity retailers and large customers cannot rely only on bilateral contracts to fulfill their electricity demand in real time. When time is close, say 1 day before real-time energy delivery, market participants may find differences between short-term load forecasts and the amount of energy procured from bilateral transactions. They need to have a market to purchase or sell energy to maintain the energy balance. Spot market is designed for energy trading in short term, which is close to the time of energy delivery. Generation companies, retailers, and large customers can buy or sell energy in day-ahead spot market and hourly-ahead spot market.

The day-ahead spot market is open to market participants 1 day before. Sellers and buyers provide their offers and bids as well as the amount of energy desired to be traded for the 24-hours of next day. The hourly-ahead spot market opens 1 hour before the real-time energy delivery. It provides a market platform for even shorter-term energy trading.

The procedures and methodologies adopted for spot market clearing are different in different markets. Basically, spot markets are cleared according to the supply curve of sellers and the demand curve of buyers. The spot market price is obtained for each hour. The total energy cleared within the hour includes energy traded in bilateral transactions/financial market and energy traded in spot markets. The detailed clearing procedure will be described in Chapter 8. In most electricity markets, Power Exchange and the ISO are responsible for energy traded through bilateral transactions, financial markets, and spot markets.

In real time, electricity demand fluctuates. The total energy traded through bilateral transactions and spot markets most probably do not perfectly match the electricity load within the hour. Real-time power balancing is needed in electricity markets. Real-time power balancing is provided as an ancillary service. In Europe, the TSO has the responsibility to obtain power in electricity balancing market to maintain real-time power balance. In the United States, the real-time power balancing is maintained by frequency regulation service and reserve service, which are procured by the ISO. Balancing service, frequency regulation, and reserves are categorized as ancillary services in the deregulated power system. They are procured in ancillary service markets. Ancillary service market is a different type of market compared to energy market. The procurement of balancing services and regulation services differs in different markets. Their pricing mechanisms are different as well.

Energy balancing requirement within each hour is satisfied by combing energy transactions of different timescales: long-term bilateral contracts, short-term spot market transactions, and real-time balancing services. The timescale of energy transaction is similar to the timescale of traditional generation scheduling.

7.5.4 Spot market pricing mechanisms

In most electricity markets, major energy transactions are through bilateral contracts. The value or price of energy within a specific hour is determined in the spot market. Electricity spot price shows the market price of energy for each hour. In the settlement of bilateral contracts and financial markets, spot price is very important. It represents the market value of the energy during the hour. In some markets, congestion costs and congestion charges are calculated on the basis of electricity spot prices.

Although every electricity market has spot market, the clearing and pricing mechanisms for each spot market could be different due to various aspects. In terms of market clearing, electricity markets can be categorized as uniform price-based markets and nodal price-based markets.

In a uniform price-based market, the spot market is cleared at the highest offer price/lowest bid price among all selected sellers/buyers. All selected sellers, no matter what their offer prices are, will be paid at the market clearing uniform price, which is higher or equal to their offer prices. All selected buyers, no matter what their bid prices are, will pay at the market clearing uniform price, which is lower or equal to their bid prices. The design of uniform price mechanism is to encourage sellers/buyers to offer/bid at prices close to their costs, hence guarantee the fairness of the market and a sustainable market operation. In some markets, due to constraints of transmission lines, the system is separated into multiple price zones. Within each price zone, a uniform market price is obtained and apply to all participants in the zone. Two neighboring zones will have different uniform prices, if there are transmission congestions between two zones. The uniform price obtained for the zone is called *zonal price*. Electricity markets in Nordic countries and some European countries adopted zonal pricing mechanisms. Without congestions, a uniform price is obtained for the whole system. When congestion occurs, the system is separated into multiple price zones, which are constrained by the transmission limits on the boarders. Uniform zonal price is obtained for each zone. Details of uniform zonal pricing will be described in chapter 8.

In a nodal price-based market, the spot market is cleared at the marginal price for each node. Due to the complexity and nonlinearity of power systems, the marginal price, which is the sensitivity of the total cost to the power change on a node, differs for different nodes. If line transmission constraints are taken into account, the differences in marginal prices between the two terminal nodes of the constrained line could be big. The marginal price is obtained for each node in the market clearing. Each node has a nodal price for every hour in the spot market. Buyers pay the nodal prices of their nodes. Similar, sellers are paid at the nodal prices of the nodes they are connected. Nodal prices cleared in the spot market are very important benchmarks (or references) for the settlement of bilateral contracts, financial contracts, as well as congestion charges and transmission rights.

The design of nodal price-based market clearing mechanisms is able to assign the costs caused by losses and congestions to market participants on the basis of cost sensitivity analysis. Nodal prices are obtained by solving the optimization model of the spot market. Given physical transmission network and transmission capacities as the constraints of the market optimization problem, the nodal prices obtained by solving the optimization problem inherently have the components that can indicate the impacts of power network and congestion constraints on the market clearing. The nodal price has three components: energy component, loss component, and congestion component. In an ideal system without losses and congestions, all nodes in the system have the same nodal price. This is similar to the uniform price. The loss component of a nodal price shows the loss contributions to the market from the market participants at the node. They need to pay for the losses caused by them. The congestion component of a nodal price shows the amount of congestion costs the participants at the node should pay for due to the amount of transmission congestions caused by their transactions.

Uniform zonal pricing and nodal pricing are two major pricing mechanisms adopted by the designers of electricity markets in the world. Deciding which pricing mechanism should be adopted depends on the system, network characteristics, and other factors. Owing to the differences of pricing mechanisms, other market issues, such as procurement of ancillary services, management of congestions, and settlement procedures are very different in zonal price-based market and nodal price-based market. In chapter 8, we will provide detailed descriptions and discussions about zonal price-based market and nodal price-based market, respectively.

7.5.5 Congestion management

Power grid is not an ideal network. Power flows following physical laws, and there is transmission capacity limit for each line. Transactions in the electricity market are subject to physical transmission restrictions. Energy transactions are executed only if the energy delivery through the grid is feasible. When market participants sign bilateral contracts or bid in spot markets, most likely they ignore the issue of transmission. The system operator is responsible for energy delivery and reliable operation. If market transactions constrain transmission capacity limits, transmission congestions occur. Economic mechanisms are necessary for successfully managing congestions in the electricity market. Both zonal price-based and nodal price-based markets have their methods to manage transmission congestions.

The zonal pricing is actually designed to handle congestion issues. The nodal pricing has a sophisticated mechanism to handle congestions as well as value congestion costs. In a zonal price-based market, when congestion occurs on tie lines, the system is separated into zones while the constrained tie line lies between zones. The power flow on the tie line is constrained by its transmission limit, which is a boundary condition for separated price zones. For each zone, the uniform price is obtained for the participants in the zone subject to the tie lie limit as boundary conditions. The zonal price of the zone at the sending end of the tie line is lower than the zonal price of the zone at the receiving end. In other words, customers in the receiving side zones have to pay a higher price due to the transmission limit that cheaper energy cannot be fully transmitted. The difference between two zonal prices is an economic signal/index that shows the value of congestions.

Nodal price is obtained by solving the market optimization model. The nodal price itself has a congestion component. The congestion component is the sensitivity or dual of the system cost to each line constraint. It provides detailed information of the impacts of line limits and congestions to the market. Market participants connected to the node, no matter sellers or buyers, pay for congestion fees according to the congestion component of the nodal price. The cost raised by transmission congestion is therefore allocated to market participants according to the contributions of their transactions to system congestion. In the United States, transmission right is derived from congestion component of nodal price as part of the compensation to transmission services. Details and calculations for congestion management will be introduced in chapter 9.

7.5.6 Transmission services

Energy is delivered through the transmission grid, which is owned and operated by the transmission company. Transmission services are provided by the transmission company to support the implementation of energy transactions. Transmission services are defined as the services provided using transmission facilities and equipment owned by the transmission companies. The services procured from generation suppliers are categorized as ancillary services, such as regulations, reserves, blackstart, and so on. The categorization of services depends on the service provider, for example, reactive power support and voltage control services. If the reactive power and voltage control service are provided by generators and synchronous condensers, it is categorized as ancillary service and can be compensated in the ancillary service market. If the reactive power and voltage control service is provided by capacitors or SVC devices in the transmission grid, it is categorized as transmission services.

Transmission companies are responsible for transmission grid construction, expansion, and maintenance. In a restructured power system, transmission is a monopoly and run by the transmission company. The costs for transmission expansion and maintenance should be recovered and charged to energy transactions. There are different ways for transmission charges. In some countries, transmission tariffs determined by regulators are charged to market participants according to their locations, voltage levels, and so on. Transmission congestions are taken into account in the spot market clearing process. When congestion occurs, the price differences of spot prices represent the congestion costs. In some markets, transmission rights and other financial tools are designed on the basis of nodal prices for hedging the risk of transmission congestions and the high cost caused by congestions. Transmission tariffs and transmission revenues are the concerns of transmission companies in the electricity market design. The regulator is responsible for the design of transmission tariffs and congestion charges.

7.6 Summary

Power system deregulation started in 1990s. Many countries have restructured their power industry since then. Electricity market designs are different in different countries and systems. In terms of products traded, there are energy market, ancillary service market, and electricity financial market. The transactions in the energy market are implemented mainly through bilateral contracts, day-ahead spot market, and hourly-ahead spot market. Spot markets are cleared with uniform zonal pricing mechanism or nodal pricing mechanism. Power system network and transmission limits are considered as the constraints in the market clearing process. Transmission congestions are charged according to the price differences of different locations. Market participants include generators, transmission companies, distribution companies, retailers, the system operator, marketers, customers, and so on. Bulk power is traded in the electricity wholesale market. Small customers purchase electricity from electricity retailers at electricity retail markets.

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CHAPTER EIGHT

Electricity market pricing models

8.1 Introduction

The costs of electric power generation depend on generation types, fuel costs, generation cost functions, and so on. In the market, electricity is not paid at its cost. Electricity has its market value at different locations during different time periods. The market value of electricity depends on the sufficiency of power supply, electricity demand of the hour, transmission availability of the network, as well as the prices offered by generators. The value of electricity is reflected by the market price of each hour. In an area with high electricity demand and insufficient power supply, the electricity has a higher value, which results in a higher electricity price. Usually, the electricity price of load center is higher than neighboring areas due to transmission restrictions. In terms of locations, electricity market is cleared for zones or cleared on the basis of nodes. Spot market prices are categorized as zonal based market price and nodal based market price. Irrespective of whether the market is priced in terms of zones or nodes, the principle of market clearing is to obtain the marginal price.

In this chapter, we will introduce market clearing mechanisms for both nodal price-based market and zonal price-based market. To illustrate electricity market clearing for a nodal price-based market, we will present the market model mathematically with an optimization problem. The optimization problem is solved to obtain locational marginal price (LMP), which is the marginal price at each node. Then, settlement for the market is discussed. Zonal price-based market and its congestion management will be illustrated at the end of this chapter.

8.2 Nodal price-based market model

8.2.1 Marginal price introduction

In commodity markets, the market price is determined by supply and demand functions. A product is priced according to its marginal production cost. Similar, in power systems, the marginal cost of power generation reflects the value of electric power. As discussed in Chapters 3 and 4, generation cost of a unit depends on its operating point (or generation output level). At different output levels, marginal costs are different. This is because of the nonlinear characteristics of generation cost curves. For a generating unit, its marginal generation cost is the derivative of generation cost function $C(P_G)$ to generation output P_G, which is $\mathrm{d}C(P_G)/\mathrm{d}P_G$. The physical meaning of marginal cost is the amount of increased generation cost $C(P_G+\Delta P_G)-C(P_G)$ if generation output increases by ΔP_G.

The calculation of marginal cost/marginal price for a system is more complicated, as there are multiple generators, also power network need to be considered. Power network is a mathematically nonlinear system. Due to line impedances and transmission limits, the cost effectiveness of demand changes on each node is different. In other words, increasing the same amount of demand ΔP_D at two different nodes may result in different system cost increases. For example, to supply one more unit demand ΔP_D at a node close to a hydro power plant, the increased cost ΔC is smaller than supply of the same amount of demand at the load center, which is far from less expensive generators. So, the system marginal cost $\Delta C/\Delta P_D$ differs for demand at different nodes. Similar, the marginal cost $\Delta C/\Delta P_G$ for generation adjustment at different nodes have different values. So, the marginal cost is obtained for each node in a power system considering its network topology, branch impedances, and transmission limits. Subject to network constraints, marginal costs at different nodes could have different values. In this way, the marginal cost is calculated for each node individually, which is a node/location-based marginal cost.

In an electricity market, cost functions are confidential information for generation companies. Generation costs are not known to the system operator. The system and market are operated based on the prices offered by generators and the prices bid by customers. The supply curve in an electricity market is constituted by all generator offer prices in an ascending order. The demand curve is constituted by all bids from customers in a descending order. However, electricity market cannot be simply cleared as other commodity markets by finding the intersection of supply curve and demand curve. This is because of the restrictions of network configurations. The sensitivity of each node to the total payment in the market is different. Marginal price of the market is calculated according to locations. It is called locational marginal price (LMP), which is a nodal price. The locational marginal price is obtained by solving the optimal power flow (OPF)-based market model.

8.2.2 Optimal power flow-based market model

Mathematical model of OPF has been introduced in Chapter 5. Traditionally, OPF is used for system economic dispatch to decide optimal generation schedule by minimizing the total generation cost in an integrated system subject to network constraints. Traditional power system is operated with central dispatch. Security-constrained OPF or security-constrained unit commitment (SCUC) are the mathematic models used by the system operator for generation scheduling.

In a nodal price-based market, the independent system operator (ISO) runs the OPF model with market inputs, including bilateral transactions, offers, and bids in the spot market. It is different from traditional central dispatch that the economic input is generation cost function of each generating unit. The OPF-based market model optimizes the total market payment while satisfying power system operation constraints: network constraints, power flow equations, generation limits, transmission limits, and so on.

8.2.2.1 Objective functions Market participants submit price–quantity pairs to the spot market. Generators with lower offers and customers with higher bids have higher priorities to be selected. The selected transactions should satisfy transmission network restrictions. The market model is formulated mathematically as an optimization problem.

The objective function of the market model is to maximize the market benefit. In other words, it is to maximize the amount paid by buyers/customers and minimize the amount paid to sellers/generators. So, the buyers with higher bids and sellers with lower offers are selected. The objective function is formulated as Equation 8.1.

$$\text{Max Benefit} = \sum_{i=1}^{N} B_i\left(P_{Di}\right) - \sum_{i=1}^{N} S_i\left(P_{Gi}\right) \tag{8.1}$$

where:

$B_i(P_{Di})$ is the aggregated demand curve constituted by bid price–quantity pairs submitted by buyers

$S_i(P_{Gi})$ is the aggregated supply curve constituted by offer price–quantity pairs submitted by sellers

Here, we have an assumption that the bids and offers can be aggregated into a continued demand curve and supply curve, respectively. However, generators usually offer piecewise prices following the piecewise linear generation curves, as we have discussed in Chapter 4. If price offers and bids are considered discrete, the objective function is formulated as in Equation 8.2.

$$\text{Max Benefit} = \sum_{i=1}^{N} \text{Bid}_i \cdot P_{Di} - \sum_{i=1}^{N} \sum_{s=1}^{Si} \text{Offer}_i^s \cdot P_{Gi}^s \tag{8.2}$$

where:

$i = 1, \ldots, N$ is the node index

Si is the number of sections that generator at node i has offered for the piecewise curve

The price offered by generator i for section s $(s = 1, \ldots, Si)$ is Offer_i^s. It is assumed that the customer at node i submits one bid, which is Bid_i.

In objective functions Equations 8.1 and 8.2, it is assumed that electricity demand is elastic demand in the market. Buyers' bids change at different load levels. However, in some cases, buyers' bids are inelastic. Then, the objective function can be formulated as follows:

$$\text{Min Payment} = \sum_{i=1}^{N} S_i\left(P_{Gi}\right) \tag{8.3}$$

Or

$$\text{Min Payment} = \sum_{i=1}^{N} \sum_{s=1}^{Si} \text{Offer}_i^s \cdot P_{Gi}^s \tag{8.4}$$

In this book, we will use Equations 8.1 and 8.3 as objective functions of the market model to study market clearing and the characteristics of locational marginal prices.

8.2.2.2 Constraints The objective function Equation 8.3 is optimized subject to system operation constraints, which are similar to the constraints of OPF model.

Power flow constraints:

$$P_{Gi} - P_{Di} + P_{Gi}^{\text{Bilateral}} - P_{Di}^{\text{Bilateral}} = \sum_{k=1}^{N} |V_i||V_k|\left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) \quad \forall i = 1,\ldots,N \tag{8.5}$$

$$Q_{Gi} - Q_{Di} = \sum_{k=1}^{N} |V_i||V_k|\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) \quad \forall i = 1,\ldots,N \tag{8.6}$$

Generation limits:

$$P_{Gi}^{\min} \le P_{Gi} + P_{Gi}^{\text{Bilateral}} \le P_{Gi}^{\max} \quad \forall i = 1,\ldots,N \tag{8.7}$$

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \quad \forall i = 1,\ldots,N \tag{8.8}$$

Bus voltage limits:

$$V_i^{\min} \le V_i \le V_i^{\max} \quad \forall i = 1,\ldots,N \tag{8.9}$$

Phase angle limits:

$$\theta_i^{\min} \le \theta_i \le \theta_i^{\max} \quad \forall i = 1,\ldots,N \tag{8.10}$$

Transmission line limits:

$$|S_{ik}|^2 \le \left(S_{ik}^{\max}\right)^2 \quad \forall i,k = 1,\ldots,N \text{ and } i \ne k \tag{8.11}$$

$$S_{ik} = P_{ik} + jQ_{ik} \quad \forall i,k = 1,\ldots,N \text{ and } i \ne k \tag{8.12}$$

$$P_{ik} = |V_i||V_k|\left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) - |V_i|^2 G_{ik} \quad \forall i,k = 1,\ldots,N \text{ and } i \ne k \tag{8.13}$$

$$Q_{ik} = |V_i||V_k|\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) - |V_i|^2\left(B_{ik} - 0.5B_{ik}^{\text{charging}}\right) \quad \forall i,k = 1,\ldots,N \text{ and } i \ne k \tag{8.14}$$

where:

- S_{ik} is the complex power that flows from node i to node k. The transmitted power on a line should be lower than its transmission capability limit
- P_{ik} is the active power flows from node i to node k
- Q_{ik} is the reactive power from node i to node k

In this model, P_{Di} is assumed to be inelastic demand. The objective function Equation 8.3 is optimized subject to constraints Equations 8.5 through 8.14. P_{Gi} and Q_{Gi} are control variables. P_{Gi} is the active power committed from the generator at node i in the spot market for the hour. As energy transactions are composed of bilateral transactions and the transactions in the spot market, the total generation at node i is the sum of P_{Gi} and $P_{Gi}^{\text{Bilateral}}$, where $P_{Gi}^{\text{Bilateral}}$ is the amount of power committed by generator i in bilateral contracts for the hour. Similar definition is for $P_{Di}^{\text{Bilateral}}$. The model Equations 8.3, 8.5 through 8.14 discussed in this section are for studying spot market clearing of energy market, so active power procured from both bilateral contracts and the spot market for the operating hour is combined as the generation output in Equations 8.5 and 8.7. In fact, there could be long-term contracts for generators providing reactive power as ancillary services. To focus on active power and energy market in this section, we simplify the reactive power procurement procedure and ignore reactive power contracts in Equations 8.6 and 8.8. Reactive power market issue will be discussed in the later chapters as one of ancillary service markets.

8.2.3 Security-constrained optimal power flow-based market model

Security operation is one of the most important issues in power systems. In any electricity market, safety and reliability are still the fundamental requirements to ensure a successful market operation. In a power system, there are possibilities that more than one lines or components are out of services during the operation. Outages of power components could result in contingency states. It is expected that the system can stand for the contingency states if one or more lines/components are lost. This is referred to as the $N-1$ criterion or $N-k$ criterion. The system operator tends to obtain generation schedules that can maintain the system within operational limits for both normal operation states and contingency states. For example, the generation schedule made by the system operator satisfies the operation requirements during normal states (without loss of lines), and even if any one of the lines is lost, the operation limits are still satisfied. This generation schedule is considered being able to stand for $N-1$ criterion. Contingency states can be extended to $N-2$ criterion, say, loss of any two components in the grid, and further to $N-3, \ldots, N-k$, and so on.

Generation schedules are traditionally obtained by solving the optimization model of OPF with operational limits and transmission limits. The system operator can make a generation schedule that is able to stand for any $N-1$ contingency state by including all $N-1$ contingencies in the constraints of the optimization model. If needed, even $N-2$ criterion could be considered. By including contingency constraints and transmission constraints, security operation requirements are achieved while optimizing the OPF model, which leads to a security-constrained optimal power flow (SCOPF) model.

In the electricity market with central dispatch, which is a common dispatch mode in many systems, SCOPF, security-constrained economic dispatch (SCED), or SCUC is used for spot market clearing. So, the transactions and the locational marginal prices settled by solving these security-constrained dispatch models have already taken system security operations into account. In the following subsections, we use SCOPF as an example to explain the security-constrained market model considering contingencies.

8.2.3.1 Objective functions The objective functions of the security-constrained market model are similar as described in Section 8.2.2.1. Variables P_{Gi} and Q_{Gi} in the SCOPF model that considers contingencies should satisfy the operation limits for both the normal state (base case) and contingency states. The objective function is optimized subject to operational limits of full network (base case) and the operational limits of the network with any contingency. In this chapter, the contingency set C includes all $N-1$ contingency c, where $c \in C$.

8.2.3.2 Constraints The constraints for the network operation for base case are same as in Section 8.2.2.2. The constraints for the network operation at contingency states include operation limits of all contingencies in the contingency set C.

Constraints for the base case:

Equations 8.5 through 8.14.

Constraints for contingency state c, $\forall c \in C$:

$$P_{Gi} - P_{Di} + P_{Gi}^{\text{Bilateral}} - P_{Di}^{\text{Bilateral}} = \sum_{k=1}^{N} \left|V_i^c\right| \left|V_k^c\right| \times \left(G_{ik}^c \cos\theta_{ik}^c + B_{ik}^c \sin\theta_{ik}^c \right) \quad \forall i = 1,\ldots,N,\ \forall c \in C \tag{8.15}$$

$$Q_{Gi} - Q_{Di} = \sum_{k=1}^{N} \left|V_i^c\right| \left|V_k^c\right| \left(G_{ik}^c \sin\theta_{ik}^c - B_{ik}^c \cos\theta_{ik}^c \right) \quad \forall i = 1,\ldots,N,\ \forall c \in C \tag{8.16}$$

$$V_i^{\min} \le V_i^c \le V_i^{\max} \quad \forall i = 1,\ldots,N,\ \forall c \in C \tag{8.17}$$

$$\theta_i^{\min} \le \theta_i^c \le \theta_i^{\max} \quad \forall i = 1,\ldots,N, \forall c \in C \tag{8.18}$$

$$\left|S_{ik}^c\right|^2 \le \left(S_{ik}^{c,\max}\right)^2 \quad \forall i,k = 1,\ldots,N \text{ and } i \ne k, \forall c \in C \tag{8.19}$$

$$S_{ik}^c = P_{ik}^c + jQ_{ik}^c \quad \forall i,k = 1,\ldots,N \text{ and } i \ne k, \forall c \in C \tag{8.20}$$

$$P_{ik}^c = \left|V_i^c\right|\left|V_k^c\right|\left(G_{ik}^c \cos\theta_{ik}^c + B_{ik}^c \sin\theta_{ik}^c\right) - \left|V_i^c\right|^2 G_{ik}^c \tag{8.21}$$

$$Q_{ik}^c = \left|V_i^c\right|\left|V_k^c\right|\left(G_{ik}^c \sin\theta_{ik}^c - B_{ik}^c \cos\theta_{ik}^c\right) - \left|V_i^c\right|^2 \times \left(B_{ik}^c - 0.5B_{ik}^{c,\text{charging}}\right) \tag{8.22}$$

where superscript c indicates that variables and parameters are for contingency state c; for example, G_{ik}^c and B_{ik}^c are the branch parameters of the network at contingency state c.

The above-mentioned formulation implements preventive security constraints. The control variables P_{Gi} and Q_{Gi} are not allowed to change for base case and contingency states. The objective function optimizes the control variables only for the base case. If using a corrective security formulation, the changes of control variables after the contingency are taken into account. In the constraints Equations 8.5 through 8.22, AC power flow is formulated. In practice, we can use DC power flow equations for network constraints. This will reduce the calculation burden significantly, although the accuracy is acceptable.

Constraints Equations 8.5 through 8.14 are the limits of base case. Constraints Equations 8.15 through 8.22 are associated with contingency states, for example loss of a line. By solving the optimization problem satisfying operation limits of the base case and all contingency states, the obtained generation schedule (or market plan) is supposed to be able to stand for any contingency state in the contingency set C. Then, the number of constraints in the security-constrained problem becomes large and depends on the size of the contingency set C. Even if DC load flow is used, the size of constrains of the problem could be very large. For example, for a system with around 2,000 branches, the number of $N-1$ contingency is around 2,000; however, the number of constraints could be several million. In practical, not all contingencies result in operation limit violations. Some contingencies only affect limited local areas.

To reduce the size and computation burden of the optimization problem, the system operator can determine the contingencies to be included in the contingency set. To decide which contingencies should be added in the security-constrained market model, the probabilities of the contingencies and the consequences of the contingencies are counted. Some critical $N-2$ contingencies could also be included.

8.2.4 Bilateral contract formulation in the market model

In an electricity market, the energy transactions of each hour are composed by energy traded through long-term contracts and energy traded in day-ahead (DA) and hourly-ahead (HA) spot markets. Real-time balancing of energy is settled in balancing market or regulation market. Spot market clearing obtains the spot price for each hour. The objective function of the market model is to maximize the market benefit or minimize the payment, as described in Section 8.2.2.1. The control variables optimized in the objective function are the generation procured in the spot market from each generator i, which is P_{Gi}. However, the generation of generator i during the hour is not only the generation procured in the spot market but also the energy traded through long-term contracts, such as bilateral contracts, over-the-counter (OTC) contracts, futures, forward, and so on for that hour. We use $P_{Gi}^{\text{Bilateral}}$ to represent the amount of energy signed by generator i through long-term/bilateral contracts for the hour. The summation of $P_{Gi}^{\text{Bilateral}}$ (through long-term contracts) and P_{Gi} (procured in the spot market) is the total generation of generator i in the hour. $P_{Gi} + P_{Gi}^{\text{Bilateral}}$ represents the generation at node i, and it is the active power generation in the power flow constraints. In the spot market model, the market clearing optimizes the payment of P_{Gi}, which is the energy procured in the spot market. The price of bilateral energy $P_{Gi}^{\text{Bilateral}}$ is not in the objective function for optimization. The reason is that long-term bilateral contracts are signed before the spot market. The price of a bilateral contract is between the seller and the buyer and not disclosed to the third party. The bilateral contract price is independent of the spot market price, and similar for prices of OTC and futures contracts. Market participants who signed bilateral contracts are responsible for informing the system operator about the amount of energy that will be traded for each hour according to their bilateral agreements. They are not required to tell the system operator of the energy price signed in the contract. The system operator includes the amount of bilateral energy in the power flow constraints to satisfy the physical limits of power network operation while optimizing the offers and bids in spot market to clear the market. In the spot market clearing procedure, only the amount of bilateral transactions is considered in the power flow calculation, the prices of bilateral transactions are independent, and they are not included in the optimization model of market clearing. In other words, the spot market clearing only use the amount of power in the bilateral contracts (and other long-term financial contracts) for checking the physical limits of network operation. If the security-constrained market model is applied, the security check for bilateral transactions is implemented as well. It is not necessary to disclose the prices of bilateral transactions to the spot market and the system operator. Similarly, only the amount of futures, forward, and other financial contracts is included in the constraints, whereas the prices of these electricity financial contracts are not shown in the spot market clearing model, although the delivery of the financial contract is on the basis of spot market price.

8.3 Locational marginal price

Marginal cost has been commonly used to price the value of the commodity in a market. In traditional economic dispatch, system marginal cost determines the optimal operation points for generators, as well as the system marginal price. For a generator, the marginal cost (or incremental cost) function is the first derivative of its generation cost function to generation output, as described in Chapter 4. If transmission losses are ignored and there is no transmission congestion, an optimized generation schedule results in equal marginal cost for each generator. In other words, the marginal cost is the same for the whole network if ignoring losses and congestions. The economic dispatch problem considering full network is described as an OPF model in Chapter 5. Due to network topology and branch impedances, generation changes at different locations may result in different incremental costs. The sensitivities of the system total cost to generation changes at different locations are different, and the sensitivities are associated with network parameters. For the OPF model Equations 5.5 through 5.10 presented in Chapter 5, the sensitivity (or dual) of total cost Equation 5.5 to active power flow constraint Equation 5.6 for each node i can be obtained, the sensitivity/dual is the marginal cost of active power for node i. If the sensitivity is calculated for the total cost to reactive power flow constraint, then the marginal cost for reactive power for each node is obtained. The sensitivity analysis method can be used for the electricity market model to obtain locational marginal prices (LMPs).

8.3.1 Formulation of locational marginal price

The optimization model for spot market has already been presented in Section 8.2.2. Depending on generator offer curves and customer bids, different objective functions Equations 8.1 through 8.4 are presented. Objective functions Equations 8.2 and 8.4 are formulated for piecewise offer prices, so the supply and demand functions are noncontinuous curves. If there are enough number of market participants for auctions, the noncontinuous supply and demand curves can be fitted to quadratic functions using curve-fitting methods. By curve fitting, the derivative of the equivalent nonlinear quadratic function can be obtained for marginal price calculation. For continuous objective functions Equations 8.1 and 8.3, the calculation of locational marginal price is more straightforward. In this section, to derive the formulation of location marginal price, for simplicity, we will use objective function Equation 8.3, Min Payment $= \sum_{i=1}^{N} S_i(P_{Gi})$.

If we use general optimization formulation, the market model that optimizes objective function Equation 8.3 subject to Equations 8.5 through 8.14 can be formulated as

Objective:

$$\text{Min}\, f(\mathbf{u}) \tag{8.23}$$

subject to:

$$\mathbf{g}(\mathbf{u}, \mathbf{x}) = \mathbf{0} \tag{8.24}$$

$$\mathbf{h}(\mathbf{u}, \mathbf{x}) \le \mathbf{0} \tag{8.25}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max} \tag{8.26}$$

where:

g is the vector of equality constraints
h is the vector of inequality constraints
u represents the vector of control variables
x represents the vector of state variables

The market optimization model is a nonlinear programming (NLP) problem. It is continuously differentiable in its feasible region. Thus, an optimality, the Karush–Kuhn–Tucker necessary condition holds:

$$\frac{\partial L}{\partial \mathbf{u}} = \mathbf{0} \tag{8.27}$$

$$\frac{\partial L}{\partial \mathbf{x}} = \mathbf{0} \tag{8.28}$$

$$\boldsymbol{\mu}^{\mathrm{T}}\mathbf{h} = \mathbf{0} \tag{8.29}$$

where $\boldsymbol{\mu}$ represents the vector of Lagrange multipliers associated with all inequality constraints $\mathbf{h}$, and L is the Lagrange function of the optimization problem that is expressed as

$$\begin{aligned}
L = & \sum_{i=1}^{N} S_i\left(P_{Gi}\right) + \sum_{i=1}^{N} \lambda_{Pi}\left[P_i - \sum_{k=1}^{N}|V_i||V_k|\left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right)\right] \\
& + \sum_{i=1}^{N} \lambda_{Qi}\left[Q_i - \sum_{k=1}^{N}|V_i||V_k|\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right)\right] \\
& + \sum_{i=1}^{N} \mu_{Pi}^{\min}\left(P_{Gi}^{\min} - P_{Gi}^{\mathrm{Bilateral}} - P_{Gi}\right) + \sum_{i=1}^{N} \mu_{Pi}^{\max}\left(P_{Gi} + P_{Gi}^{\mathrm{Bilateral}} - P_{Gi}^{\max}\right) \\
& + \sum_{i=1}^{N} \mu_{Qi}^{\min}\left(Q_{Gi}^{\min} - Q_{Gi}\right) + \sum_{i=1}^{N} \mu_{Qi}^{\max}\left(Q_{Gi} - Q_{Gi}^{\max}\right) \\
& + \sum_{i=1}^{N} \mu_{Vi}^{\min}\left(|V_i|^{\min} - |V_i|\right) + \sum_{i=1}^{N} \mu_{Vi}^{\max}\left(|V_i| - |V_i|^{\max}\right) \\
& + \sum_{i=1}^{N} \mu_{\theta i}^{\min}\left(\theta_i^{\min} - \theta_i\right) + \sum_{i=1}^{N} \mu_{\theta i}^{\max}\left(\theta_i - \theta_i^{\max}\right) \\
& + \sum_{i=1}^{N}\sum_{k=1}^{N} \mu_{ik}\left[\begin{array}{l}
\left(|V_i||V_k|\left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) - |V_i|^2 G_{ik}\right)^2 \\
+\left(|V_i||V_k|\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right)\right. \\
\left.-|V_i|^2\left(B_{ik} - 0.5B_{ik}^{\mathrm{charging}}\right)\right)^2 - \left(S_{ik}^{\max}\right)^2
\end{array}\right]
\end{aligned} \tag{8.30}$$

where:

P_i is the net injection of active power at node i. $P_i = P_{Gi} - P_{Di} + P_{Gi}^{\text{Bilateral}} - P_{Di}^{\text{Bilateral}}$ as expressed in Equation 8.5

Q_i is the net injection of reactive power at node i. $Q_i = Q_{Gi} - Q_{Di}$ as expressed in Equation 8.6

The locational marginal price at node i, LMP_i, which represents the incremental payment of the system in supplying a marginal demand at node i, is expressed as the Lagrange multiplier of the active power balance of node i:

$$\text{LMP}_i = \lambda_{Pi} \tag{8.31}$$

The locational marginal price (LMP) at each node i is composed of three components: energy component, loss component, and congestion component.

Physically speaking, the energy component represents the marginal value of supplying electricity in an ideal network without losses or transmission limits. Energy components are same for all nodes in the network. It is an indication of the value of energy. In Chapter 4, economic dispatch without considering network obtains a system-wide marginal cost. In fact, this is the same principle as the energy component of LMP.

Loss component represents the additional cost caused by transmission losses. Owing to network topology structure and line impedances, losses associated with different nodes are different. Loss component of each node depends on its location and the sensitivity of system losses to active power flow constraint. The economic dispatch considering losses in Chapter 4 applies a penalty factor pf_i (Section 4.3) as a correction of losses to the system marginal cost for each node i. Its principle is similar as that of the loss component of LMP.

Congestion component is a nonzero value only if there are transmission congestions in the network, or power flow on a line reaches the transmission limit of the line. For example, there is a two-area system that power transfer from area A to area B, and generation cost in A is mostly lower than that in B. If the transmission line between A and B has an unlimited capacity, power can be transmitted freely to B and then the energy price will be the same for two areas. Given the line between A and B has a transmission limit T^{Max}, the power that flows from A to B is limited to T^{Max}. If more power than T^{Max} is needed in B, then area B has to purchase energy from its local generators, which is more expensive than purchase from area A. So, the energy price of area B will be higher than that of area A. The price difference of the two areas is caused by the capacity limit of transmission line (or transmission congestion). This is the congestion component. In a messed network, the calculation of congestion component is not as straightforward as in the two-area example. Mathematical tools are needed to calculate the congestion component of LMP.

There are different ways to decompose LMP into energy component, loss component, and congestion component. The methods can be classified into two categories: reference node-dependent methods and reference node-independent methods. When people first started to decompose a LMP into three components, it was noticed that some system parameters are given depending upon the selection of reference node, such as slack bus in power flow calculation. So, the decomposition of a LMP is referred to the selected reference bus, which needs to be specified. Using different reference buses may result in different values for the components. However, the summation of three components is the LMP, which cannot be changed. The difference between congestion components of two nodes should always be the same no matter which reference bus is used, and the difference represents the congestion value/congestion charge between two nodes.

The component values obtained by the classical LMP decomposition method are reference dependent. The reference node needs to be specified when three components are presented. Some reference-independent methods proposed by researchers can be found in the recent research papers. In practical market analysis, we care about the value of LMP and the differences in loss components between nodes and differences of congestion components between nodes, which are relative values that do not depend on reference nodes. Thus, absolute values and reference selections do not matter that much. Here, we introduce the classical LMP decomposition method.

Classical LMP decomposition method is based on the classical power flow analysis and Jacobian matrix. The slack bus is selected as the reference bus. The voltage magnitude and phase angle are fixed for the slack bus.

Because of network topology and branch impedances, some energy is consumed in the network. So, the total generation from generators is larger than the total electricity demand in the network. If the energy consumed in the network is P_{loss}, then

$$P_{\text{loss}} = \sum_{i=1}^{N}\left(P_{Gi} + P_{Gi}^{\text{Bilateral}}\right) - \sum_{i=1}^{N}\left(P_{Di} + P_{Di}^{\text{Bilateral}}\right)$$

where:

P_{Gi} and P_{Di} are the power cleared in the spot market

$P_{Gi}^{\text{Bilateral}}$ and $P_{Di}^{\text{Bilateral}}$ are the power traded through long-term transactions

If constraints Equations 8.5 and 8.6 are represented by net active power injection P_i and net reactive power injection Q_i as follows

$$\begin{aligned} P_i &= P_{Gi} - P_{Di} + P_{Gi}^{\text{Bilateral}} - P_{Di}^{\text{Bilateral}} \\ &= \sum_{k=1}^{N} |V_i||V_k|\left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) \quad \forall i = 1,\ldots,N \end{aligned} \tag{8.32}$$

$$Q_i = Q_{Gi} - Q_{Di} = \sum_{k=1}^{N} |V_i||V_k|\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) \quad \forall i = 1,\ldots,N \tag{8.33}$$

then we have

$$\begin{aligned} P_{\text{loss}} &= \sum_{i=1}^{N}\left(P_{Gi} + P_{Gi}^{\text{Bilateral}}\right) - \sum_{i=1}^{N}\left(P_{Di} + P_{Di}^{\text{Bilateral}}\right) \\ &= \sum_{i=1}^{N}\left(P_{Gi} - P_{Di} + P_{Gi}^{\text{Bilateral}} - P_{Di}^{\text{Bilateral}}\right) \\ &= \sum_{i=1}^{N} P_i \quad \forall i = 1,\ldots,N \end{aligned} \tag{8.34}$$

If the net power injection to the reference bus is listed separately and the index of the reference bus is called as ref, then Equation 8.34 is expressed as

$$P_{\text{loss}} = \sum_{\substack{i=1 \\ i \neq \text{ref}}}^{N} P_i + P_{\text{ref}} \tag{8.35}$$

From Equations 8.32 and 8.33, net power injections P_i and Q_i are functions of state variables V_i and θ_i. In Equations 8.23 through 8.26, state variables are represented by the variable vector $\mathbf{x}$, and $\mathbf{x} = [V_1, V_2, \ldots, V_N, \theta_1, \theta_2, \ldots, \theta_N]^T$. So, P_i, Q_i, P_{loss}, and P_{ref} all are functions of the state variable $\mathbf{x}$, expressed as follows

$$P_i = P_i\left(V_1, V_2, \ldots, V_N, \theta_1, \theta_2, \ldots, \theta_N\right) \text{ or } P_i = P_i(\mathbf{x})$$

$$Q_i = Q_i\left(V_1, V_2, \ldots, V_N, \theta_1, \theta_2, \ldots, \theta_N\right) \text{ or } Q_i = Q_i(\mathbf{x})$$

$$P_{\text{loss}} = P_{\text{loss}}\left(V_1, V_2, \ldots, V_N, \theta_1, \theta_2, \ldots, \theta_N\right) \text{ or } P_{\text{loss}} = P_{\text{loss}}(\mathbf{x})$$

$$P_{\text{ref}} = P_{\text{ref}}\left(V_1, V_2, \ldots, V_N, \theta_1, \theta_2, \ldots, \theta_N\right) \text{ or } P_{\text{ref}} = P_{\text{ref}}(\mathbf{x})$$

According to Equation 8.35, $P_{\text{loss}}(\mathbf{x})$ can be written as

$$P_{\text{loss}}(\mathbf{x}) = \sum_{\substack{i=1 \\ i \neq \text{ref}}}^{N} P_i(\mathbf{x}) + P_{\text{ref}}(\mathbf{x}), \text{ so } P_{\text{ref}}(\mathbf{x}) = P_{\text{loss}}(\mathbf{x}) - \sum_{\substack{i=1 \\ i \neq \text{ref}}}^{N} P_i(\mathbf{x}) \tag{8.36}$$

The partial derivative $P_{\text{ref}}(\mathbf{x})$ with respect to state variable: $\mathbf{x} = [V_1, V_2, \ldots, V_N, \theta_1, \theta_2, \ldots, \theta_N]^T$ is

$$\frac{\partial P_{\text{ref}}(\mathbf{x})}{\partial \mathbf{x}} = \frac{\partial P_{\text{loss}}(\mathbf{x})}{\partial \mathbf{x}} - \frac{\partial \sum_{\substack{i=1 \\ i \neq \text{ref}}}^{N} P_i(\mathbf{x})}{\partial \mathbf{x}} = \frac{\partial P_{\text{loss}}(\mathbf{x})}{\partial \mathbf{x}} - \sum_{\substack{i=1 \\ i \neq \text{ref}}}^{N} \frac{\partial P_i(\mathbf{x})}{\partial \mathbf{x}} \tag{8.37}$$

As P_{loss} is also a function of control variables P_{Gi} and Q_{Gi} (for $i = 1, \ldots, N$, and $i \neq \text{ref}$), hence P_i and Q_i, so

$$\frac{\partial P_{\text{loss}}(\mathbf{x})}{\partial \mathbf{x}} = \sum_{\substack{i=1 \\ i \notin \text{ref}}}^{N} \frac{\partial P_{\text{loss}}}{\partial P_i} \cdot \frac{\partial P_i(\mathbf{x})}{\partial \mathbf{x}} + \sum_{\substack{i=1 \\ i \notin \text{ref}}}^{N} \frac{\partial P_{\text{loss}}}{\partial Q_i} \cdot \frac{\partial Q_i(\mathbf{x})}{\partial \mathbf{x}} \tag{8.38}$$

Replace $\partial P_{\text{loss}}(\mathbf{x})/\partial \mathbf{x}$ in Equation 8.37 by Equation 8.38, then Equation 8.37 is formulated as

$$\begin{aligned}\frac{\partial P_{\text{ref}}(\mathbf{x})}{\partial \mathbf{x}} &= \frac{\partial P_{\text{loss}}(\mathbf{x})}{\partial \mathbf{x}} - \sum_{\substack{i=1\\ i\neq \text{ref}}}^{N} \frac{\partial P_i(\mathbf{x})}{\partial \mathbf{x}} \\ &= \sum_{\substack{i=1\\ i\notin \text{ref}}}^{N} \frac{\partial P_{\text{loss}}}{\partial P_i}\cdot\frac{\partial P_i(\mathbf{x})}{\partial \mathbf{x}} + \sum_{\substack{i=1\\ i\notin \text{ref}}}^{N} \frac{\partial P_{\text{loss}}}{\partial Q_i}\cdot\frac{\partial Q_i(\mathbf{x})}{\partial \mathbf{x}} \\ &\quad - \sum_{\substack{i=1\\ i\neq \text{ref}}}^{N} \frac{\partial P_i(\mathbf{x})}{\partial \mathbf{x}} \\ &= \sum_{\substack{i=1\\ i\notin \text{ref}}}^{N} \left(\frac{\partial P_{\text{loss}}}{\partial P_i} - 1\right)\cdot\frac{\partial P_i(\mathbf{x})}{\partial \mathbf{x}} + \sum_{\substack{i=1\\ i\notin \text{ref}}}^{N} \frac{\partial P_{\text{loss}}}{\partial Q_i}\cdot\frac{\partial Q_i(\mathbf{x})}{\partial \mathbf{x}}\end{aligned} \tag{8.39}$$

Equation 8.39 can be expanded and expressed with matrix formulations as follows:

$$\frac{\partial P_{\text{ref}}(\mathbf{x})}{\partial \mathbf{x}} = \left[\frac{\partial P_1(\mathbf{x})}{\partial \mathbf{x}},\ldots,\frac{\partial P_N(\mathbf{x})}{\partial \mathbf{x}},\frac{\partial Q_1(\mathbf{x})}{\partial \mathbf{x}},\ldots,\frac{\partial Q_N(\mathbf{x})}{\partial \mathbf{x}}\right] \times \begin{bmatrix}\left(\frac{\partial P_{\text{loss}}}{\partial P_1} - 1\right)\\ \vdots \\ \left(\frac{\partial P_{\text{loss}}}{\partial P_N} - 1\right)\\ \frac{\partial P_{\text{loss}}}{\partial Q_1}\\ \vdots\\ \frac{\partial P_{\text{loss}}}{\partial Q_N}\end{bmatrix} \quad \text{For } i = 1,\ldots,N, \text{ and } i \neq \text{ref} \tag{8.40}$$

Vector $[\partial P_1(\mathbf{x})/\partial \mathbf{x},\ldots,\partial P_N(\mathbf{x})/\partial \mathbf{x},\partial Q_1(\mathbf{x})/\partial \mathbf{x},\ldots,\partial Q_N(\mathbf{x})/\partial \mathbf{x}]$ is the transpose of Jacobian matrix $\mathbf{J}$ if we apply the definition of Jacobian matrix in power system analysis, so

$$\mathbf{J}^T = \left[\frac{\partial P_1(\mathbf{x})}{\partial \mathbf{x}},\ldots,\frac{\partial P_N(\mathbf{x})}{\partial \mathbf{x}},\frac{\partial Q_1(\mathbf{x})}{\partial \mathbf{x}},\ldots,\frac{\partial Q_N(\mathbf{x})}{\partial \mathbf{x}}\right]$$

Formula (8.40) is written as

$$\frac{\partial P_{\text{ref}}(\mathbf{x})}{\partial \mathbf{x}} = \mathbf{J}^T \cdot \begin{bmatrix} \left(\frac{\partial P_{\text{loss}}}{\partial P_1} - 1\right) \\ \vdots \\ \left(\frac{\partial P_{\text{loss}}}{\partial P_N} - 1\right) \\ \frac{\partial P_{\text{loss}}}{\partial Q_1} \\ \vdots \\ \frac{\partial P_{\text{loss}}}{\partial Q_N} \end{bmatrix}, \quad \text{or}$$

$$\frac{\partial P_{\text{ref}}(\mathbf{x})}{\partial \mathbf{x}} = -\mathbf{J}^T \cdot \begin{bmatrix} \left(1 - \frac{\partial P_{\text{loss}}}{\partial P_1}\right) \\ \vdots \\ \left(1 - \frac{\partial P_{\text{loss}}}{\partial P_N}\right) \\ -\frac{\partial P_{\text{loss}}}{\partial Q_1} \\ \vdots \\ -\frac{\partial P_{\text{loss}}}{\partial Q_N} \end{bmatrix} \quad \begin{matrix} \text{For } i = 1,\ldots,N, \\ \text{and } i \neq \text{ref} \end{matrix} \tag{8.41}$$

We can also write the partial derivative of Lagrange function (Equation 8.30) with respect to the state variable vector **x** as Equation 8.42

$$\frac{\partial L}{\partial \mathbf{x}} = \frac{\partial \sum_{i=1}^{N} S_i(P_{Gi})}{\partial \mathbf{x}} + \sum_{i=1}^{N} \lambda_{Pi} \frac{\partial P_i}{\partial \mathbf{x}} + \sum_{i=1}^{N} \lambda_{Qi} \frac{\partial Q_i}{\partial \mathbf{x}} + \frac{\partial \mathbf{h}^T}{\partial \mathbf{x}} \mu \tag{8.42}$$

Or

$$\frac{\partial L}{\partial \mathbf{x}} = \frac{\partial \sum_{i=1}^{N} S_i(P_{Gi})}{\partial \mathbf{x}} + \frac{\partial \mathbf{g}^{\mathrm{T}}}{\partial \mathbf{x}} \lambda + \frac{\partial \mathbf{h}^{\mathrm{T}}}{\partial \mathbf{x}} \mu$$

where:

g is the vector of equality constraints

h is the vector of inequality constraints

μ is the vector of Lagrange multipliers associated with all the inequality constraints **h**

λ is the vector of Lagrange multipliers associated with equality constraints **g**

$$\boldsymbol{\lambda} = \left[\lambda_{P1}, \ldots, \lambda_{PN}, \lambda_{Q1}, \ldots, \lambda_{QN}\right]^T$$

Component λ_{Pi} is the locational marginal price for active power at bus i. If we study reactive power pricing, then, in fact, λ_{Qi} is the locational marginal price for reactive power at bus i.

Similar as the derivation in Equations 8.40 and 8.41, the second and third items in Equation 8.42 associated with Lagrange multipliers λ_{Pi} and λ_{Qi} can be represented as the product of Jacobian matrix $\mathbf{J}^T$ and vector of Lagrange multiplier $\boldsymbol{\lambda}$. So, Equation 8.42 is written as

$$\frac{\partial L}{\partial \mathbf{x}} = \frac{\partial \sum_{i=1}^{N} S_i(P_{Gi})}{\partial \mathbf{x}} + \mathbf{J}^T \boldsymbol{\lambda} + \frac{\partial \mathbf{h}^T}{\partial \mathbf{x}} \boldsymbol{\mu} \tag{8.43}$$

The necessary condition for optimality is that the derivatives of Lagrange function to variables are equal to zero, which is $(\partial L / \partial \mathbf{x}) = \mathbf{0}$. Then, we can obtain the formulation of vector $\boldsymbol{\lambda}$, which is LMP as follows:

$$\boldsymbol{\lambda} = -(\mathbf{J}^T)^{-1} \left(\frac{\partial \sum_{i=1}^{N} S_i(P_{Gi})}{\partial \mathbf{x}} + \frac{\partial \mathbf{h}^T}{\partial \mathbf{x}} \boldsymbol{\mu} \right)$$

$$= -(\mathbf{J}^T)^{-1} \frac{\partial \sum_{i=1}^{N} S_i(P_{Gi})}{\partial \mathbf{x}} - (\mathbf{J}^T)^{-1} \frac{\partial \mathbf{h}^T}{\partial \mathbf{x}} \boldsymbol{\mu}$$

The formulation is further written as

$$\boldsymbol{\lambda} = -(\mathbf{J}^T)^{-1}\left(\frac{\partial \sum_{i=1}^{N} S_i(P_{Gi})}{\partial P_{\text{ref}}} \cdot \frac{\partial P_{\text{ref}}}{\partial \mathbf{x}}\right) - (\mathbf{J}^{\text{T}})^{-1}\frac{\partial \mathbf{h}^T}{\partial \mathbf{x}}\boldsymbol{\mu}$$

$$= -(\mathbf{J}^T)^{-1}\left(\frac{\partial \sum_{\substack{i=1 \\ i\neq \text{ref}}}^{N} S_i(P_{Gi})}{\partial P_{\text{ref}}} \cdot \frac{\partial P_{\text{ref}}}{\partial \mathbf{x}} + \frac{\partial S_{\text{ref}}(P_{\text{ref}})}{\partial P_{\text{ref}}} \cdot \frac{\partial P_{\text{ref}}}{\partial \mathbf{x}}\right) \tag{8.44}$$

$$-(\mathbf{J}^T)^{-1}\frac{\partial \mathbf{h}^T}{\partial \mathbf{x}}\mu$$

$$= -(\mathbf{J}^T)^{-1}\left(\frac{\partial S_{\text{ref}}(P_{\text{ref}})}{\partial P_{\text{ref}}} \cdot \frac{\partial P_{\text{ref}}}{\partial \mathbf{x}}\right) - (\mathbf{J}^T)^{-1}\frac{\partial \mathbf{h}^T}{\partial \mathbf{x}}\boldsymbol{\mu}$$

By substituting Equations 8.41 into 8.44, we have

$$\lambda = -(\mathbf{J}^T)^{-1}\left(\frac{\partial S_{\text{ref}}(P_{\text{ref}})}{\partial P_{\text{ref}}} \cdot \mathbf{J}^T \cdot \begin{bmatrix} \left(\frac{\partial P_{\text{loss}}}{\partial P_1} - 1\right) \\ \vdots \\ \left(\frac{\partial P_{\text{loss}}}{\partial P_N} - 1\right) \\ \frac{\partial P_{\text{loss}}}{\partial Q_1} \\ \vdots \\ \frac{\partial P_{\text{loss}}}{\partial Q_N} \end{bmatrix}\right) \tag{8.45}$$

$$-(\mathbf{J}^T)^{-1}\frac{\partial \mathbf{h}^T}{\partial \mathbf{x}}\mu$$

As $(\mathbf{J}^T)^{-1} \cdot \mathbf{J}^T = \mathbf{1}$, then

$$\lambda = -\left(\frac{\partial S_{\text{ref}}\left(P_{\text{ref}}\right)}{\partial P_{\text{ref}}} \cdot \begin{bmatrix} \left(\dfrac{\partial P_{\text{loss}}}{\partial P_1} - 1\right) \\ \vdots \\ \left(\dfrac{\partial P_{\text{loss}}}{\partial P_N} - 1\right) \\ \dfrac{\partial P_{\text{loss}}}{\partial Q_1} \\ \vdots \\ \dfrac{\partial P_{\text{loss}}}{\partial Q_N} \end{bmatrix} \right) - (\mathbf{J}^T)^{-1} \frac{\partial \mathbf{h}^T}{\partial \mathbf{x}} \mu \tag{8.46}$$

From the vector formulation of Equation 8.46, we can find that, for any bus i, the LMP λ_{Pi} is

$$\lambda_{Pi} = \frac{\partial S_{\text{ref}}(P_{\text{ref}})}{\partial P_{\text{ref}}} - \frac{\partial P_{\text{loss}}}{\partial P_i} \cdot \frac{\partial S_{\text{ref}}(P_{\text{ref}})}{\partial P_{\text{ref}}} - \frac{\partial \mathbf{h}^T}{\partial P_i} \mu \tag{8.47}$$

It is shown in Equation 8.47 that the LMP at bus i is composed of three components. The first component $\partial S_{\text{ref}}(P_{\text{ref}})/\partial P_{\text{ref}}$ is the energy component, which is related to the marginal cost at the reference node. The second component $-(\partial P_{\text{loss}}/\partial P_i) \cdot ([\partial S_{\text{ref}}(P_{\text{ref}})]/\partial P_{\text{ref}})$ is the loss component. The last component $-(\partial \mathbf{h}^T/\partial P_i)\boldsymbol{\mu}$ is the congestion component.

The energy component is independent of bus index, which means the energy component is same for all nodes in the grid. However, the value relies on the selection of reference node. We have discussed this issue in the above sections. The loss component is the product of energy component and the derivative of system loss to P_i, which means that loss component differs at different buses. The congestion component is associated with the partial differential of inequality constraints to P_i, which represents the costs caused by constrained transmission limits.

8.3.2 Derivation of locational marginal price for a security-constrained market model

SCOPF or SCUC is commonly adopted by the ISO as market model. Locational marginal prices are obtained from security-constrained market model. The derivation of LMP for SCOPF model will be described here. Same as in the previous section, we first write the Lagrange equation for the security-constrained market model (Equations 8.3, 8.5 through 8.22), as in Equation 8.48.

$$
\begin{aligned}
L = & \sum_{i=1}^{N} S_i(P_{Gi}) + \sum_{i=1}^{N} \lambda_{Pi}\left[P_{Gi} - \sum_{k=1}^{N} |V_i||V_k|\left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right)\right] \\
& + \sum_{i=1}^{N} \lambda_{Qi}\left[Q_{Gi} - \sum_{k=1}^{N} |V_i||V_k|\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right)\right] \\
& + \sum_{i=1}^{N} \mu_{Pi}^{\min}\left(P_{Gi}^{\min} - P_{Gi}^{\text{Bilateral}} - P_{Gi}\right) + \sum_{i=1}^{N} \mu_{Pi}^{\max}\left(P_{Gi} + P_{Gi}^{\text{Bilateral}} - P_{Gi}^{\max}\right) \\
& + \sum_{i=1}^{N} \mu_{Qi}^{\min}\left(Q_{Gi}^{\min} - Q_{Gi}\right) + \sum_{i=1}^{N} \mu_{Qi}^{\max}\left(Q_{Gi} - Q_{Gi}^{\max}\right) \\
& + \sum_{i=1}^{N} \mu_{Vi}^{\min}\left(|V_i|^{\min} - |V_i|\right) + \sum_{i=1}^{N} \mu_{Vi}^{\max}\left(|V_i| - |V_i|^{\max}\right) \\
& + \sum_{i=1}^{N} \mu_{\theta i}^{\min}\left(\theta_i^{\min} - \theta_i\right) + \sum_{i=1}^{N} \mu_{\theta i}^{\max}\left(\theta_i - \theta_i^{\max}\right) \\
& + \sum_{i=1}^{N}\sum_{k=1}^{N} \mu_{ik}\left[\begin{array}{l} \left(|V_i||V_k|\left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) - |V_i|^2 G_{ik}\right)^2 + \\ \left(|V_i||V_k|\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) - \right. \\ \left. \quad |V_i|^2\left(B_{ik} - 0.5B_{ik}^{\text{charging}}\right)\right)^2 - \left(S_{ik}^{\max}\right)^2 \end{array}\right] \\
& + \sum_{c\in \boldsymbol{C}}\sum_{i=1}^{N} \lambda_{Pi}^{c}\left[P_{Gi} - \sum_{k=1}^{N} |V_i^c||V_k^c|\left(G_{ik}^c\cos\theta_{ik}^c + B_{ik}^c\sin\theta_{ik}^c\right)\right] \\
& + \sum_{c\in C}\sum_{i=1}^{N} \lambda_{Qi}^{c}\left[Q_{Gi} - \sum_{k=1}^{N} |V_i^c||V_k^c|\left(G_{ik}^c\sin\theta_{ik}^c - B_{ik}^c\cos\theta_{ik}^c\right)\right] \\
& + \sum_{c\in C}\left[\sum_{i=1}^{N} \mu_{Vi}^{\min,c}\left(|V_i|^{\min} - |V_i^c|\right) + \sum_{i=1}^{N} \mu_{Vi}^{\max,c}\left(|V_i^c| - |V_i|^{\max}\right)\right] \\
& + \sum_{c\in C}\left[\sum_{i=1}^{N} \mu_{\theta i}^{\min,c}\left(\theta_i^{\min} - \theta_i^c\right) + \sum_{i=1}^{N} \mu_{\theta i}^{\max,c}\left(\theta_i^c - \theta_i^{\max}\right)\right] \\
& + \sum_{c\in C}\sum_{i=1}^{N}\sum_{k=1}^{N} \mu_{ik}^{c}\left[\begin{array}{l} \left(|V_i^c||V_k^c|\left(G_{ik}^c\cos\theta_{ik}^c + B_{ik}^c\sin\theta_{ik}^c\right) - |V_i^c|^2 G_{ik}^c\right)^2 + \\ \left(|V_i^c||V_k^c|\left(G_{ik}^c\sin\theta_{ik}^c - B_{ik}^c\cos\theta_{ik}^c\right) - \right. \\ \left. \quad |V_i^c|^2\left(B_{ik}^c - 0.5B_{ik}^{c,\text{charging}}\right)\right)^2 - \left(S_{ik}^{c,\max}\right)^2 \end{array}\right]
\end{aligned}
\tag{8.48}
$$

LMP is obtained as the derivative of Lagrange function to active power P_i at bus i. The LMP considering contingency states can be expressed as

$$\mathrm{LMP}_i = \frac{\partial L}{\partial P_i} = \lambda_{Pi} + \sum_{c \in C} \lambda_{Pi}^c \tag{8.49}$$

The LMP has two items: (1) λ_{Pi} is the same as the LMP procured for the OPF model, representing the locational marginal price in the precontingency state and (2) $\sum_{c \in C} \lambda_{Pi}^c$ representing the locational marginal cost in meeting the power balance equations for all contingency scenarios. Therefore, $\sum_{c \in C} \lambda_{Pi}^c$ is defined as the marginal price for contingency set C. From this aspect, security can be priced in the electricity market.

8.4 Examples for locational marginal price calculation and market clearing

In this section, we will use several examples to illustrate how to clear the spot market and how to obtain locational marginal prices, as well as how to integrate bilateral contracts in the market settlement, and the applications of contract for difference (CfD) in the market.

8.4.1 An example of locational marginal price calculation without loss or congestion

8.4.1.1 Example A A simple example is given here to illustrate the LMP calculation without considering network. Assume there are three generators (power suppliers) in the market, and the system load P_D is inelastic load. They offer linear supply functions, so the offer prices are constant numbers as shown in Table 8.1.

Without considering the network effect of losses and congestions, the market model is quite simple. It can be formulated as follows:

$$\text{Min Payment} = \sum_{i=1}^{N} S_i(P_{Gi}) = 12P_{G1} + 15P_{G2} + 20P_{G3} \tag{8.50}$$

Table 8.1 Generator offer prices for Example A

Generator	Capacity (MW)	Offer price (\$/MWh)
Gen 1	300	12
Gen 2	150	15
Gen 3	200	20

Subject to

$$P_{G1} + P_{G2} + P_{G3} = P_D \tag{8.51}$$

$$0 \le P_{G1} \le 300 \tag{8.52}$$

$$0 \le P_{G2} \le 150 \tag{8.53}$$

$$0 \le P_{G3} \le 200 \tag{8.54}$$

The market model (Equations 8.50 through 8.54) has a discontinuous objective function due to the piecewise payment function. The optimality can be obtained for each section. The marginal price is λ, which is formulated as

$$\lambda = \begin{cases} 12 & 0 < P_D \le 300 \\ 15 & 300 < P_D \le 450 \\ 20 & 450 < P_D \le 650 \end{cases}$$

As network effect is not considered, there is no difference identified for the locations of three generators. LMP is the same for all generators, and equal to λ. In fact, this is the energy component, as no loss or congestion component is counted.

For the example, if the load is 400 MW, it falls in the second section, the marginal price is $15/MWh for selected two generators: Generator 1 and Generator 2. Generator 3 is not selected due to its high offer price. If the load is 500 MW, the demand falls in the third section, the marginal price is $20/MWh for three generators, which are all selected by the market.

8.4.2 Examples of locational marginal price calculation with network effects

In this section, we use a three-node system to illustrate the effects of network congestions on LMP and the calculation of congestion component. It is assumed that resistances of lines are zero, and the power flow distribution is based on line impedances. We can simply use DC load flow equations for power flow calculation.

8.4.2.1 Examples: Without loss or congestion

8.4.2.1.1 Example B1 The topology of a three-node power system is same as shown in Figure 8.1. Resistance of each line is ignored, which means there is no loss considered. The reactance of each line is given in the figure.

There are two generators located at node 1 and node 2, respectively. The capacity of the generator at node 1, G1, is 300 MW, and its offer price is $10/MWh. The capacity of the generator

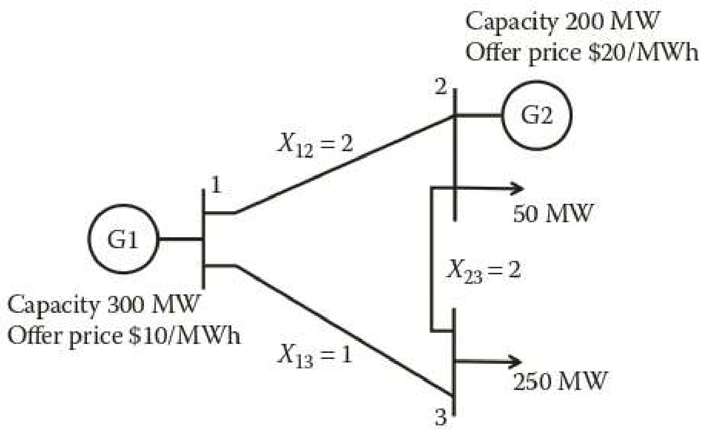

FIGURE 8.1 The three-node system: load = 300 MW and no congestion.

at node 2, G2, is 200 MW, and its offer price is $20/MWh. The total load is 300 MW on node 2 and node 3, where $P_{D2} = 50$ MW and $P_{D3} = 250$ MW.

Assuming that transmission line limits are not constrained in this example, there is no congestion. In this example, only generation offers are considered in the market. DC OPF introduced in Chapter 5 is applied, and the DC power flow equation is used for the power balance constraint for each node. The market model of this example can be written as follows:

$$\text{Min Payment} = \sum_{i=1}^{3} S_i(P_{Gi}) = 10P_{G1} + 20P_{G2} \tag{8.55}$$

Subject to

DC power flow constraints:

$$P_{Gi} - P_{Di} = \sum_{\substack{k=1 \\ k \neq i}}^{N} \frac{(\theta_i - \theta_k)}{X_{ik}} \quad \forall i = 1, \ldots, 3$$

So,

$$P_{G1} = \frac{(\theta_1 - \theta_2)}{2} + \frac{(\theta_1 - \theta_3)}{1} \tag{8.56}$$

$$P_{G2} - P_{D2} = \frac{(\theta_2 - \theta_1)}{2} + \frac{(\theta_2 - \theta_3)}{2} \tag{8.57}$$

$$-P_{D3} = \frac{(\theta_3 - \theta_1)}{1} + \frac{(\theta_3 - \theta_2)}{2} \tag{8.58}$$

Generation limits:

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \quad \forall i = 1,\ldots,3$$

So,

$$0 \le P_{G1} \le P_{G1}^{\text{Max}} \tag{8.59}$$

$$0 \le P_{G2} \le P_{G2}^{\text{Max}} \tag{8.60}$$

Define base value $S_B = 100\,\text{MVA}$, so per unit value of P_{D2} is 50/100 = 0.5, and per unit value of P_{D3} is 250/100 = 2.5. The per unit value for generator capacity is 3 p.u. for Generator 1 and 2 p.u. for Generator 2. Rewriting the model (Equations 8.55 through 8.60), we have a simple optimization model as follows:

$$\text{Min Payment} = 10P_{G1} + 20P_{G2} \tag{8.61}$$

Subject to

$$P_{G1} = 1.5\theta_1 - 0.5\theta_2 - \theta_3 \tag{8.62}$$

$$P_{G2} - 0.5 = -0.5\theta_1 + \theta_2 - 0.5\theta_3 \tag{8.63}$$

$$-2.5 = -\theta_1 - 0.5\theta_2 + 1.5\theta_3 \tag{8.64}$$

$$0 \le P_{G1} \le 3 \tag{8.65}$$

$$0 \le P_{G2} \le 2 \tag{8.66}$$

The objective function of model (Equations 8.61 through 8.66) is a two-segment function. Using the Lagrange function, the optimal solution for the model can be easily obtained as $P_{G1} = 3$ (per unit), $P_{G2} = 0$. Substituting results in the model, we can simplify the original Lagrange function as follows to calculate Lagrange factors:

$$\begin{aligned} L = 10P_{G1} &+ \lambda_1\left(1.5\theta_1 - 0.5\theta_2 - \theta_3 - P_{G1}\right) \\ &+ \lambda_2\left(-0.5\theta_1 + \theta_2 - 0.5\theta_3 - P_{G2} + 0.5\right) \\ &+ \lambda_3\left(-\theta_1 - 0.5\theta_2 + 1.5\theta_3 + 2.5\right) \end{aligned}$$

where λ_1, λ_2, and λ_3 are the Lagrange factors for power balance equations (Equations 8.62 through 8.64). To obtain the optimum solution, we have

$$\frac{\partial L}{\partial P_{G1}} = 10 - \lambda_1 = 0$$

$$\frac{\partial L}{\partial \theta_1} = 1.5\lambda_1 - 0.5\lambda_2 - \lambda_3 = 0$$

$$\frac{\partial L}{\partial \theta_2} = -0.5\lambda_1 + \lambda_2 - 0.5\lambda_3 = 0$$

$$\frac{\partial L}{\partial \theta_2} = -\lambda_1 - 0.5\lambda_2 + 1.5\lambda_3 = 0$$

By solving the earlier four equations, we can obtain $\lambda_1 = \lambda_2 = \lambda_3 = \$10/\text{MWh}$, which are the LMPs of the nodes.

If we set the voltage phase angle of node 1 as reference, say $\theta_1 = 0$, and substitute $P_{G1} = 3$ and $P_{G2} = 0$ into Equations 8.62 through 8.64, the equations can be solved and the value of phase angles are obtained as $\theta_1 = 0$, $\theta_2 = -(8/5)$, and $\theta_3 = -(11/5)$. According to DC power flow calculation, the power flow between node i and j, P_{ij} (p.u. value) is expressed as $P_{ij} = (\theta_i - \theta_j)/X_{ij}$. So, we have the power flow (per unit value) on each line as follows:

$$P_{12} = \frac{(\theta_1 - \theta_2)}{X_{12}} = \frac{[0-(-8/5)]}{2} = \frac{4}{5}$$

$$P_{13} = \frac{(\theta_1 - \theta_3)}{X_{13}} = \frac{[0-(-11/5)]}{1} = \frac{11}{5}$$

$$P_{23} = \frac{(\theta_2 - \theta_3)}{X_{23}} = \frac{[(-8/5)-(-11/5)]}{2} = \frac{3}{10}$$

Converting the per unit values to actual values based on $S_B = 100$ MVA, we have

$$P_{12} = \frac{4}{5}\times 100 = 80 \text{ MW},\ P_{13} = \frac{11}{5}\times 100 = 220 \text{ MW, and}$$

$$P_{23} = \frac{3}{10}\times 100 = 30 \text{ MW},$$

$$P_{G1} = 300 \text{ MW},\ P_{G2} = 0, \text{ and } \text{LMP}_1 = \text{LMP}_2 = \text{LMP}_3 = \$10/\text{MWh}.$$

The price \$10/MWh is in fact the energy component of LMP, as there is no loss or congestion in the market. The total payment to Generator 1 is $\$10/\text{MWh} \times 300 \text{ MWh} = \$3,000$ for 1 hour.

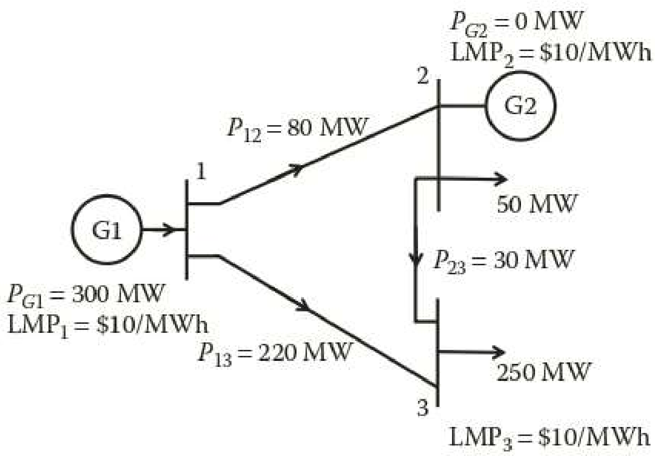

FIGURE 8.2 Solution to the three-node market model: load = 300 MW and no congestion.

Load at node 2 needs to pay $\$10/\text{MWh} \times 50\ \text{MWh} = \$500$, and load at node 3 needs to pay $\$10/\text{MWh} \times 250\ \text{MWh} = \$2{,}500$.

The solution to the example is shown in Figure 8.2.

8.4.2.1.2 Example B2 We continue to test the market model in Example B1 and assume that the load is increased to 400 MW, with $P_{D2} = 50$ MW and $P_{D3} = 350$ MW. Rewriting the model (Equations 8.61 through 8.66), we can optimize the model for Load = 400 MW (or 4 p.u.) as follows:

$$\text{Min Payment} = 10P_{G1} + 20P_{G2}$$

Subject to

$$\begin{aligned}
&P_{G1} = 1.5\theta_1 - 0.5\theta_2 - \theta_3 \\
&P_{G2} - 0.5 = -0.5\theta_1 + \theta_2 - 0.5\theta_3 \\
&-3.5 = -\theta_1 - 0.5\theta_2 + 1.5\theta_3 \qquad (8.67) \\
&0 \le P_{G1} \le 3 \\
&0 \le P_{G2} \le 2
\end{aligned}$$

The objective functions and constraints are same as in the model (Equations 8.61 through 8.66), except that Equation 8.64 is replaced by Equation 8.67 due to change in load at node 3 from 2.5 to 3.5 p.u. Using the similar method as Example B1, the model is solved and the solutions are as follows:

$$P_{12} = 40\ \text{MW},\ P_{13} = 260\ \text{MW, and } P_{23} = 90\ \text{MW}$$

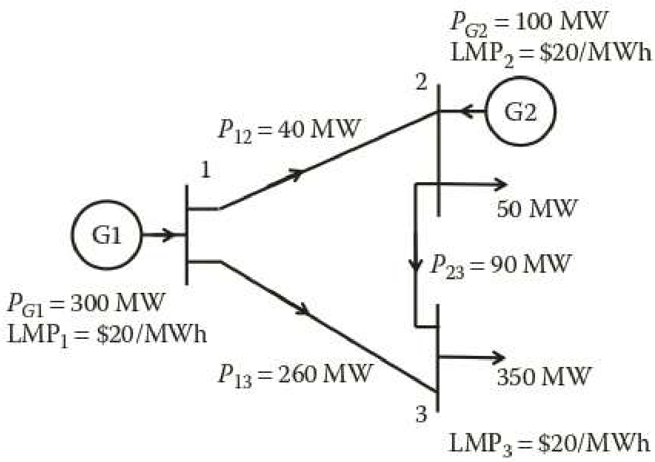

FIGURE 8.3 Solution to the three-node market model: load = 400 MW and no congestion.

$P_{G1} = 300\,\text{MW}$, $P_{G2} = 100\,\text{MW}$, and $\text{LMP}_1 = \text{LMP}_2 = \text{LMP}_3 =$ \$20/MWh

The solution for the scenario that load = 400 MW is shown in Figure 8.3.

From the market model solutions, we found that for a market without congestion, and without considering losses, the locational marginal price for each node is the same, and all are equal to the offer price of the marginal generator, which is the last generator selected for supplying the load. In Example B1, the load is 300 MW, Generator 1, which offers lower price, is selected to supply 300 MW and LMP is \$10/MWh. In Example B2, the load is 400 MW, which is higher than the capacity of Generator 1, so Generator 2 is also selected and LMP becomes \$20/MWh, which is the offer price by Generator 2. In fact, since no losses or congestions are considered in the examples, there are no loss and congestion components. So, the LMPs obtained in Examples B1 and B2 are the energy components of locational marginal prices.

8.4.2.2 Examples: With congestions

8.4.2.2.1 Example C1 In this example, we continue Example B1 and assume the line between node 1 and 3 has a transmission limit $P_{13}^{\text{Max}} = 200$ MW; so, the per unit value of the transmission limit is 2 p.u. based on $S_B = 100$ MVA. The market optimization model for this example is formulated based on the optimization model (Equations 8.61 through 8.66) in Example B1, with an additional transmission line constraint

$P_{13} = (\theta_1 - \theta_3)/X_{13} \le P_{13}^{\text{Max}}$ added to the model. The objective function and other constraints are same as in Example B1. By substituting the value of $X_{13} = 1$ and $P_{13}^{\text{Max}} = 2$ p.u., the transmission line constraint for line 1–3 is rewritten as $\theta_1 - \theta_3 \le 2$. The constraint is formulated as Equation 8.68 in the following model:

$$\text{Min Payment} = 10P_{G1} + 20P_{G2}$$

Subject to

$$P_{G1} = 1.5\theta_1 - 0.5\theta_2 - \theta_3$$

$$P_{G2} - 0.5 = -0.5\theta_1 + \theta_2 - 0.5\theta_3$$

$$-2.5 = -\theta_1 - 0.5\theta_2 + 1.5\theta_3$$

$$\theta_1 - \theta_3 \le 2 \tag{8.68}$$

$$0 \le P_{G1} \le 3$$

$$0 \le P_{G2} \le 2$$

The Lagrange function with power flow constraints and the transmission constraint is written as

$$\begin{aligned} L = 10P_{G1} + 20P_{G2} &+ \lambda_1\left(1.5\theta_1 - 0.5\theta_2 - \theta_3 - P_{G1}\right) \\ &+ \lambda_2\left(-0.5\theta_1 + \theta_2 - 0.5\theta_3 - P_{G2} + 0.5\right) \\ &+ \lambda_3\left(-\theta_1 - 0.5\theta_2 + 1.5\theta_3 + 2.5\right) \\ &+ \mu_{13}\left(\theta_1 - \theta_3 - 2\right) \end{aligned}$$

The necessary conditions for optimal solution are that the derivatives of the Lagrange function with respect to control variables and state variables are equal to zero. So,

$$\frac{\partial L}{\partial P_{G1}} = 0 \qquad 10 - \lambda_1 = 0$$

$$\frac{\partial L}{\partial P_{G2}} = 0 \qquad 20 - \lambda_1 = 0$$

$$\frac{\partial L}{\partial \theta_1} = 0 \qquad 1.5\lambda_1 - 0.5\lambda_2 - \lambda_3 + \mu_{13} = 0$$

$$\frac{\partial L}{\partial \theta_2} = 0 \qquad -0.5\lambda_1 + \lambda_2 - 0.5\lambda_3 = 0$$

$$\frac{\partial L}{\partial \theta_2} = 0 \qquad -\lambda_1 - 0.5\lambda_2 + 1.5\lambda_3 - \mu_{13} = 0$$

By solving the previous five equations, we can obtain

$\lambda_1 = \$10/\text{MWh}$, $\lambda_2 = \$20/\text{MWh}$, $\lambda_3 = \$30/\text{MWh}$, and $\mu_{13} = \$25/\text{MWh}$

The optimal solution for power generation is

$$P_{G1} = 250 \text{ MW and } P_{G2} = 50 \text{ MW}$$

$$P_{12} = 50 \text{ MW}, P_{13} = 200 \text{ MW, and } P_{23} = 50 \text{ MW}$$

The solution is shown in Figure 8.4.

The results show that locational marginal prices for three nodes become different due to the congestion between node 1 and node 3. The LMPs are $\text{LMP}_1 = \$10/\text{MWh}$, $\text{LMP}_2 = \$20/\text{MWh}$, and $\text{LMP}_3 = \$30/\text{MWh}$.

The given conditions of the example are same as in Example B1, except that a transmission limit 200 MW between node 1 and 3 is constrained. Due to the transmission limit and congestion between node 1 and 3, we notice that the optimal generation schedule and LMPs have a big difference compared

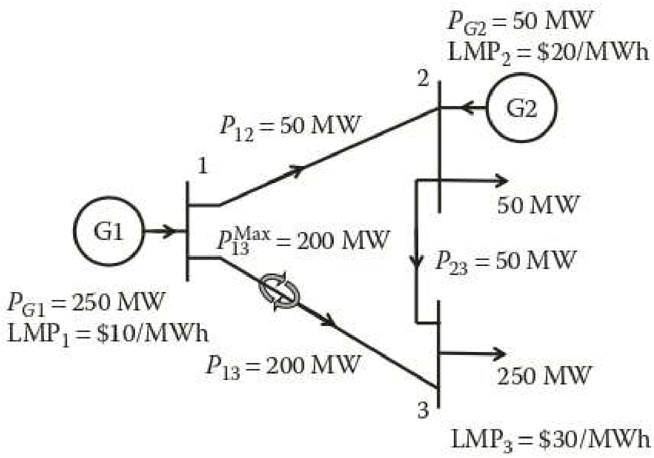

FIGURE 8.4 Solution to the three-node market model: load = 300 MW and with congestions.

to the results of Example B1, which has no transmission limit. The less expensive generator, Generator 1, is not dispatched to generate at its full capacity due to the transmission ability of line 1–3. Generator 1 generates 250 MW, and the remaining 50 MW is generated by the expensive Generator 2. Power flow is redistributed according to line impedances. Without transmission congestion, LMPs for all node are same and equal to \$10/MWh. However, with congestions, LMP at node 1 is \$10/MWh, LMP at node 2 is \$20/MWh, and LMP at node 3 is \$30/MWh. This means that customers at node 2 and node 3 will need to pay more than that shown by the results in Example B1. Load at node 2 needs to pay $\$20/\text{MWh} \times 50\ \text{MWh} = \$1,000$, and load at node 3 needs to pay $\$30/\text{MWh} \times 250\ \text{MWh} = \$7,500$. Both loads need to pay much higher than the case in Example B1. The total payment from loads is $\$1,000 + \$7,500 = \$8,500$. At the generation side, Generator 1 is paid by $\$10/\text{MWh} \times 250\ \text{MWh} = \$2,500$ and the Generator 2 is paid by $\$20/\text{MWh} \times 50\ \text{MWh} = \$1,000$. The total amount received by generators is $\$2,500 + \$1,000 = \$3,500$. The difference between the amount paid by loads and the amount received by the generators is the congestion cost, which is $\$8,500 - \$3,500 = \$5,000$. From this example, it is shown that the congestion cost could be very high, and the risk of congestion (or delivery risk) is high in an electricity market. The methods to hedge congestion risk are necessary in an electricity market. The issue of congestion management in an electricity market will be introduced in Chapter 9.

8.4.2.2.2 Example C2 To verify the locational marginal price obtained for node 3 in Example C1, we increase P_{D3} by $\Delta P_{D3} = 10$ MW to 260 MW and calculate the incremental payment due to the load increase. The following model is solved to obtain the solution for the problem:

$$\text{Min Payment} = 10P_{G1} + 20P_{G2}$$

Subject to

$$P_{G1} = 1.5\theta_1 - 0.5\theta_2 - \theta_3$$

$$P_{G2} - 0.5 = -0.5\theta_1 + \theta_2 - 0.5\theta_3$$

$$-2.6 = -\theta_1 - 0.5\theta_2 + 1.5\theta_3$$

$$\theta_1 - \theta_3 \le 2$$

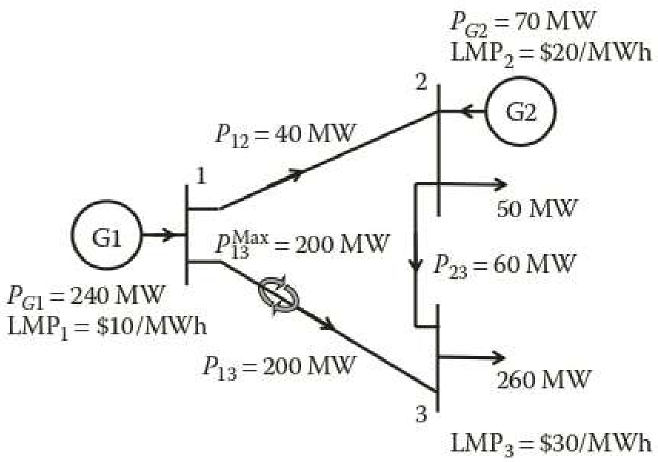

FIGURE 8.5 Solution to the three-node market model: load = 310 MW and with congestions.

The solution of the aforementioned model is as follows:

$$\lambda_1 = \$10/\text{MWh},\ \lambda_2 = \$20/\text{MWh},\ \text{and}\ \lambda_3 = \$30/\text{MWh}$$

$$P_{G1} = 240\ \text{MW and}\ P_{G2} = 70\ \text{MW}$$

$$P_{12} = 40\ \text{MW},\ P_{13} = 200\ \text{MW},\ \text{and}\ P_{23} = 60\ \text{MW}$$

The solution is shown in Figure 8.5.

The total payment to two generators is $\$10/\text{MWh} \times 240\ \text{MWh} + \$20/\text{MWh} \times 50\ \text{MWh} = \$3{,}800$. Compared to Example C1, the incremental payment due to the load increase $\Delta P_{D3} = 10$ MW is $\$3{,}800 - \$3{,}500 = \$300$. So, the marginal price is $300/10 MWh = $30/MWh, which is equal to the LMP obtained in the example. The LMP calculation for node 3 is verified.

8.4.3 Locational marginal price calculation for a system

In this section, we give an example to calculate LMP for a power system. In Chapter 5, we use the IEEE 30-bus system as the test system for OPF. Here, we will use the same system for LMP calculation (Figure 8.6). The system topology and data are given in Section 5.6.1. Following is the one-line diagram of the system and the supply function $S_i(P_{Gi})$.

The supply functions $S_i(P_{Gi})$ offered by six generators in the system are assumed to be quadratic functions instead of piecewise linear functions. The assumption makes it less complicated for calculation. $S_i(P_{Gi}) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i$, where the coefficients are listed in Table 8.2. The demand P_{Di} for bus i is listed in Table 8.3.

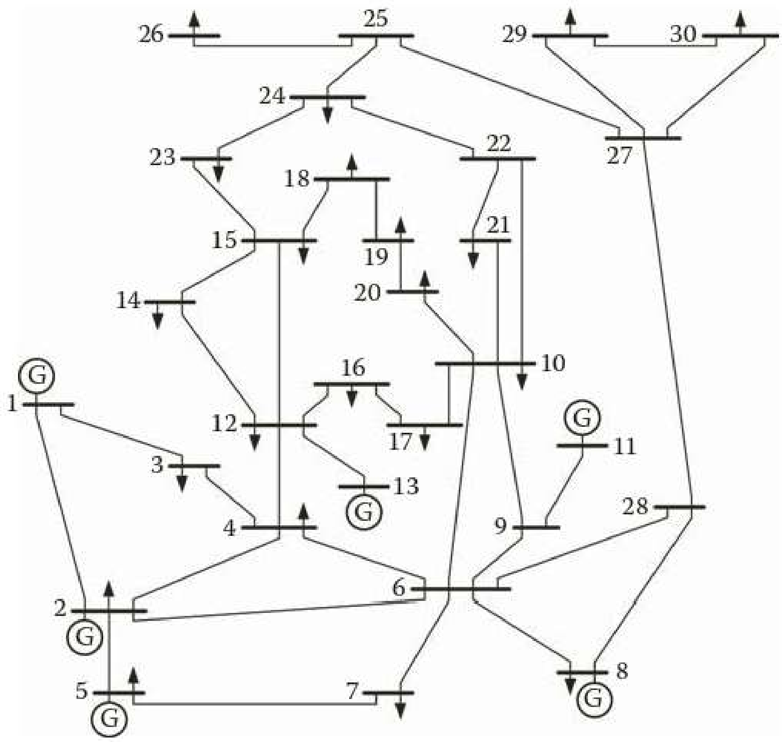

FIGURE 8.6 One-line diagram of IEEE 30-bus system.

Table 8.2 Coefficients of supply functions and upper limits of generations

Generator i	a_i ($/MWh^2)	b_i ($/MWh)	c_i ($)	P_{Gi}^{Max} (MW)
1	0.12	30	80	40
2	0.05	25	40	80
5	0.1	38	25	80
8	0.2	20	35	80
11	0.15	40	50	100
13	0.08	32	50	180

Table 8.3 Active power demand for system (unit: MW)

i	P_{Di}	i	P_{Di}	i	P_{Di}	i	P_{Di}	i	P_{Di}	i	P_{Di}
1	0	6	0	11	0	16	3.5	21	17.5	26	3.5
2	21.7	7	22.8	12	11.2	17	9	22	0	27	0
3	2.4	8	30	13	0	18	3.2	23	3.2	28	0
4	7.6	9	0	14	6.2	19	9.5	24	8.7	29	2.4
5	94.2	10	5.8	15	8.2	20	2.2	25	0	30	10.6

8.4.3.1 Example D1 We first use DC load flow-based market model to calculate the LMP without losses and congestions. The LMPs for all buses are obtained as \$43.13/MWh. As losses are ignored and there is no congestion, the LMP value \$43.13/MWh is the energy component of locational marginal price. The results are shown in Table 8.4.

In the market, the seller/generator is paid with the LMP at its node. The payment to each generator and the total payment is calculated and listed in Table 8.4.

8.4.3.2 Example D2 If we use AC load flow equations that consider transmission losses, the market schedule is obtained as given in Table 8.5. The LMP obtained for all 30 buses are listed in Table 8.6. The payment to each generator and the total payment is listed in Table 8.5.

Table 8.4 Market schedules, LMPs, and total payment without losses and congestions

Generator i	Generation schedule P_{Gi} (MW·h)	LMP_i (\$/MWh)	Payment ($\text{LMP}_i \times P_{Gi}$) (\$)
1	40	43.13	1725.2
2	80	43.13	3450.4
5	25.63	43.13	1105.4
8	57.815	43.13	2493.6
11	10.42	43.13	449.4
13	69.536	43.13	2999.1
Total	283.4	–	12223.1

Table 8.5 Market schedules considering transmission losses

Generator i	Generation schedule P_{Gi} (MW·h)	LMP_i (\$/MWh)	Payment ($\text{LMP}_i \times P_{Gi}$) (\$)
1	40	42.28	1691.2
2	80	42.60	3408
5	32.267	44.45	1434.3
8	57.887	43.15	2497.8
11	11.819	43.55	514.7
13	64.792	42.37	2745.2
Total	286.765	–	12291.2

Table 8.6 LMPs ($/MWh) for all buses considering transmission losses

i	LMP_i	i	LMP_i	i	LMP_i	i	LMP_i	i	LMP_i	i	LMP_i
◎1	42.28	6	44.33	◎11	43.55	16	43.31	21	44.04	26	44.95
◎2	42.60	7	44.02	12	42.37	17	43.64	22	44.02	27	43.90
3	42.88	◎8	43.15	◎13	42.37	18	44.07	23	43.96	28	43.49
4	43.04	9	43.55	14	43.03	19	44.30	24	44.35	29	44.93
◎5	44.45	10	43.66	15	43.43	20	44.17	25	44.26	30	45.64

LMPs for all buses are listed in Table 8.6. For the six buses with generators, a symbol ◎ is marked beside the bus number. Among six generators, generator at bus 5 is more expensive in the market, generators at bus 1, 2, and 13 are relatively less expensive. Bus 1 has the lowest LMP. Bus 30 is far from power suppliers, it has the highest LMP due to long-distance transmission and transmission losses.

According to the amount of generation and LMP at nodes with generators, the payment to each generator is calculated and shown in the last column of Table 8.5. The total payment is $12,291.2. Using the demand data given in Table 8.3 and the LMPs in Table 8.6, the total amount paid by the buyers/consumers is calculated as $\sum_{i=1}^{N} \text{LMP}_i \times P_{Di} = \$12{,}440$. The total payment according to LMPs to sellers/generators is $12,291.2 as calculated in Table 8.5. The difference between the amount charged to buyers and the payment to sellers is ($12,440 − $12,291.2) = $148.8, owning to losses.

In this example, transmission line limits are assumed to be big and there is no transmission congestion. The power flow in the branch, P_{ik}, from bus i to k $(i,k \in 1,\ldots,N,\ i \neq k)$ is listed in Table 8.7.

The value of P_{ik} in the table shows the power flow from bus i to k. For a negative power flow P_{ik}, it refers to power flow from k to i. There are three lines, line 2–5, line 6–7, and line 12–13, the power flow on the line is higher than 40 MW.

8.4.3.3 Example D3 If transmission capacities for all lines are limited by 40 MW, the power flow on lines 2–5, lines 6 and 7, and lines 12 and 13 in Example D2 needs to be redistributed. By considering transmission limits $-40 \leq P_{ik} \leq 40$, the ACOPF is calculated with losses and congestions considered. The results of generation schedules and LMPs are shown in Tables 8.8 and 8.9.

Table 8.7 Power flow P_{ik} (MW) in branches without transmission limits

From i	To k	P_{ik}	From i	To k	P_{ik}	From i	To k	P_{ik}	From i	To k	P_{ik}
1	2	21.22	6	8	−20.95	10	22	6.73	19	20	−2.46
1	3	18.78	6	9	9.33	12	13	−64.79	21	22	−3.02
2	4	14.00	6	10	7.60	12	14	9.51	22	24	3.68
2	5	45.78	6	28	8.48	12	15	25.53	23	24	6.52
2	6	19.68	8	28	6.89	12	16	15.78	24	25	1.44
3	4	16.24	9	10	21.15	14	15	3.22	25	26	3.53
4	6	25.29	9	11	−11.82	15	18	10.36	25	27	−2.10
4	12	−2.77	10	17	−3.02	15	23	9.81	27	28	−15.34
5	7	−16.97	10	20	4.69	16	17	12.08	27	29	6.18
6	7	40.25	10	21	14.55	18	19	7.06	27	30	7.05
									29	30	3.71

Table 8.8 Market schedules considering transmission losses and congestions

Generator i	Generation schedule P_{Gi} (MW·h)	LMP_i (\$/MWh)	Payment ($LMP_i \times P_{Gi}$) (\$)
1	40	44.32	1772.8
2	80	44.59	3567.2
5	43.954	46.79	2056.6
8	63.279	45.31	2867.2
11	18.829	45.65	859.5
13	40	38.40	1536
Total	286.06	–	12659.3

Table 8.9 LMPs (\$/MWh) for all buses considering transmission losses and congestions

i	LMP_i	i	LMP_i	i	LMP_i	i	LMP_i	i	LMP_i	i	LMP_i
©1	44.32	6	45.54	©11	45.65	16	45.59	21	46.12	26	47.10
©2	44.59	7	46.31	12	44.92	17	45.79	22	46.10	27	45.98
3	45.10	©8	45.31	©13	38.40	18	46.40	23	46.32	28	45.70
4	45.31	9	45.65	14	45.60	19	46.54	24	46.56	29	47.05
©5	46.79	10	45.71	15	45.88	20	46.36	25	46.39	30	47.77

The results in the tables show that the generation output of generator at bus 13 is constrained to 40 MW due to the transmission limit on the line between buses 12 and 13, which is the only line connecting Generator 13 to the rest of the network. The LMP at bus 13 decreases from \$42.37 to \$38.40 because of congestion. Table 8.9 shows that the lowest LMP is at bus 13. The congestion price between buses 12 and 13 is around (\$44.92 − \$38.40) = \$6.42, assuming losses on the line are neglectable. It shows that congestion cost is quite huge. If a seller on bus 13 and a buyer on bus 12 sign a bilateral contract, they will need to share the congestion cost. The mechanisms of how to share the cost will be discussed in Section 8.5.

If we compare the LMPs obtained without congestions in Table 8.6 and LMPs obtained with congestions in Table 8.9, we can see that LMPs are generally higher in the cases of congestions, except LMP at bus 13 is lower because its energy cannot be fully delivered due to the line congestion, and the relatively expensive generator 5 is selected to generate more electricity, and the LMP at 5 is high. Bus 30 and the surrounding buses are far from those generators, they have high LMPs, which means they need to pay more for long-distance transmission.

The results in Table 8.8 show that the total payment to sellers/generators is \$12,659.3, which is much higher that the payment (\$12,291.2) in the case without transmission limits, as shown in Table 8.5. Using the demand data given in Table 8.3 and the LMPs in Table 8.9, the total amount paid by the buyers/consumers in the case of considering congestions is calculated as $\sum_{i=1}^{N} \mathrm{LMP}_i \times P_{Di}$ = \$13,078.9. The difference between the amount charged to buyers and the payment to sellers is (\$13,078.9 − \$12,659.3) = \$419.6. This amount is due to transmission congestions and transmission losses, and the major portion is transmission congestion.

The power flow in the branch, P_{ik}, from bus i to k $(i,k \in 1,\ldots,N,\ i \neq k)$ are listed as in Table 8.10. The lines reaching the transmission upper limit 40 MW are the line between buses 2 and 5, and the line between buses 12 and 15.

8.4.3.4 Example D4 In this example, we continue with Example D2 (ACOPF considering losses) by adding security constraints and solve the AC load flow-based SCOPF market model. It is assumed that transmission line limits are big and no congestions in this example for simplicity. We will focus on the effects of security constraints and contingencies on the LMPs. The lines selected for the $N-1$ contingency set are highlighted in Figure 8.7.

Table 8.10 Power flow P_{ik} (MW) in branches for the case with transmission limits

From i	To k	P_{ik}	From i	To k	P_{ik}	From i	To k	P_{ik}	From i	To k	P_{ik}
1	2	19.27	6	8	−25.31	10	22	7.35	19	20	−5.32
1	3	20.73	6	9	10.22	12	13	−40	21	22	−2.06
2	4	16.92	6	10	9.49	12	14	8.31	22	24	5.26
2	5	40	6	28	8.82	12	15	20.66	23	24	3.58
2	6	20.59	8	28	7.91	12	16	10.29	24	25	0.09
3	4	18.17	9	10	29.05	14	15	2.04	25	26	3.53
4	6	16.84	9	11	−18.83	15	18	7.44	25	27	−3.44
4	12	10.47	10	17	2.31	15	23	6.82	27	28	−16.68
5	7	−10.87	10	20	7.57	16	17	6.71	27	29	6.18
6	7	33.98	10	21	15.51	18	19	4.19	27	30	7.05
									29	30	3.71

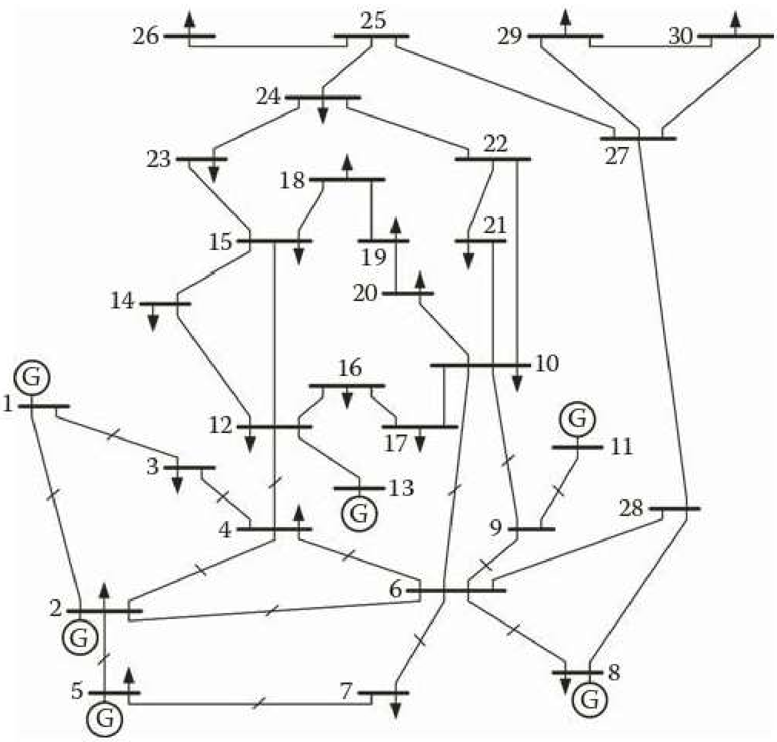

FIGURE 8.7 Highlighted contingency set for the IEEE 30-bus system.

The results of generation schedules and LMPs for the SCOPF market model are shown in Tables 8.11 and 8.12.

The results in Table 8.11 show that the generator at bus 11 is scheduled as zero. The reason is that the line between buses 9 and 11 is in the contingency set and it is the only line that

Table 8.11 Market schedules considering $N-1$ contingencies in SCOPF

Generator i	Generation schedule P_{Gi} (MW·h)	LMP_i (\$/MWh)	Payment ($\text{LMP}_i \times P_{Gi}$) (\$)
1	40	41.81	1672.4
2	80	41.95	3356
5	45.249	47.05	2129
8	58.644	43.46	2548.7
11	0	40	0
13	65.210	42.43	2766.9
Total	289.103	–	12473

Table 8.12 LMPs (\$/MWh) for all buses considering $N-1$ contingencies in SCOPF

i	LMP_i	i	LMP_i	i	LMP_i	i	LMP_i	i	LMP_i	i	LMP_i
©1	41.81	6	43.64	©11	40.00	16	43.52	21	44.38	26	45.22
©2	41.95	7	45.17	12	42.43	17	43.94	22	44.35	27	44.16
3	42.83	©8	43.46	©13	42.43	18	44.29	23	44.16	28	43.80
4	43.11	9	43.88	14	43.12	19	44.56	24	44.62	29	45.20
©5	47.05	10	44	15	43.57	20	44.46	25	44.52	30	45.91

connects Generator 11 to the rest of the network. To guarantee that generations are enough even if the line between buses 9 and 11 is out of service, Generator 11 is not dispatched to satisfy the security requirements. The relatively expensive generator at bus 5 is scheduled for more energy in the market, and the LMP at bus 5 becomes the highest in the network. In fact, it shows the value of Generator 5 in the system if contingencies are considered as security constraints.

Using the demand data given in Table 8.3 and the LMPs in Table 8.12, we can calculate the total amount paid by buyers/consumers as $\sum_{i=1}^{N} \text{LMP}_i \times P_{Di} = \$12{,}731.8$. In Example D2, the total amount paid by buyers/consumers is \$12,291.2. The difference between them is (\$12,731.8 − \$12,291.2) = \$440.6. This is the cost to maintain the system security operation for the market in case any one of the contingencies in the contingency set occurs. It can be considered as the cost for security operation. If all 41 lines are selected in the contingency set, say $N-1$ operation ($N = 41$), the cost of maintaining the security operation would be higher than that in the case study in this example, which has only 15 lines in the contingency set.

8.5 Market settlement

The spot market is cleared after the auctions in day-ahead (DA) and hourly-ahead (HA) markets. Market participants provide price–quantity pairs for the market auction. Sellers offer price–quantity pairs for electricity supply. Buyers bid price–quantity pairs for electricity consumption. In a market, the major portion of energy is traded through long-term bilateral contracts, forward, and futures markets, although the hourly price for energy, which is LMP discussed here, is determined in the spot market. In this section, we will introduce how to include bilateral energy in the spot market and how to settle long-term bilateral contracts in the spot market, since the owners of bilateral contracts need to be responsible for the losses and congestions caused by their energy transactions.

8.5.1 Bilateral contracts in a spot market

In the market model Equations 8.3, 8.5 through 8.14 presented in Section 8.2, the amount of energy traded through bilateral contracts and financial markets is included in the mathematical model. Bilateral contracts are signed for the trading of energy (MWh or kWh) for a period. The contracted energy is scheduled to be delivered on an hourly basis during the period. The system operator has been informed of the energy delivery with following information: the amount of energy delivery during a specific hour for this contract, the injection bus of the energy, and the withdraw bus of the energy. The energy price in the contract is not necessary to be disclosed to the system operator. Same ways apply to the energy traded in other financial markets, such as OTC, forward, and futures markets. As energy traded through these long-term contracts are separated into hourly based energy delivery, the quantities of the hourly-energy are included in the spot market model, and they are presented in the model as MWh quantities without the associated prices. This amount of energy is categorized as self-scheduled energy, or self-schedules. The quantities of self-schedules together with the price–quantity offers and price–quantity bids are optimized in the least-cost market dispatch model, which has been presented in Section 8.2. By solving the market model, the spot market is cleared, and LMPs for all nodes in the spot market are obtained. In the market clearing, self-schedules are settled as price-takers; their energy contract prices are not in the market clearing model. Self-scheduled energy will not be adjusted unless the market cannot be cleared using the offers and bids.

8.5.2 Bilateral contract settlement

The bilateral contracts between a seller and a buyer could be signed years before energy delivery time. The energy price in the contract is determined by the seller and buyer themselves. The quantity of the energy contract is informed to the system operator for operation purpose. When sellers and buyers negotiate the details of bilateral contracts, the issues of real system operation, such as losses and congestions, are not estimated and included. However, in real system operation, bilateral energy transactions in the network cause losses and even congestions in certain time periods. The owners of bilateral contracts should be responsible and pay for the losses and congestions resulting from their energy transactions. Also, the sellers and buyers need to decide how to allocate the loss cost and congestion cost between them.

The LMP cleared in the spot market has a loss component and a congestion component. If a seller of a bilateral contract injects energy into one node, in the spot market, the seller should be paid with the LMP of this node, while if the buyer of the contract withdraws energy from another node, the buyer should pay with the LMP of the node in the spot market. Following the payment mechanisms in the spot market, the loss and congestion costs caused by the bilateral transaction is paid to the market. Using LMPs of energy injection node and withdraw node to clear the energy injected into and withdrawn from the network makes it possible for the two parties of the transaction to pay for the losses and congestions caused by the transaction in the spot market. However, the LMPs of the two nodes have no direct relationship with the contract price signed in the bilateral contract. The bilateral contract could be signed years or months before the energy delivery, while the spot market is cleared based on offers and bids that are available hourly ahead of the energy delivery.

In the market operation, a bilateral contract by different parties is actually paid twice, we call double payment. Then, the double payment is offset in the market settlement system, or it is offset through CfD. Offsetting payments should not eliminate the costs of losses and congestions. Only the energy component is offset. For the loss and congestion cost caused by the bilateral transaction, the two parties of the transaction can determine how to allocate the cost between them. The cost allocation can be implemented by selecting different offsetting methods and different reference trading hubs. The double payment mechanism and offsetting payment methods are illustrated in the examples in the following section.

8.5.3 Examples for bilateral contract settlement

The spot market is cleared with the optimization market model presented in early sections of this chapter. LMPs are obtained by solving the market clearing model. The settlement of DA and HA spot market for market participants is straightforward. The seller who injects energy into a node will be paid with the LMP of the node. A buyer who consumes energy at a node will pay the energy according to the local LMP. By following this settlement mechanism, the losses and congestions caused by delivering energy from and to them are paid to the market, and the energy component payment is cleared as well.

However, settlement of a bilateral contract is a little bit more complicated, as the two parties of the contract have a contract price signed for the energy. They need to fulfill the contract in addition to following the spot market settlement system. In fact, this is also a method to hedge the risk of high spot market price. Bilateral contracts have the similar effects as other electricity financial tools to hedge market price risks.

In the following subsections, we will use examples to illustrate the settlement of bilateral contracts with several steps.

8.5.3.1 Example I: Double payment of bilateral contract Assume a seller S and a buyer B have signed a bilateral contract. The contract is to sell energy 100 MWh from node 1 to node 2 during a specific hour at a price of \$50/MWh. The contract is signed between S and B, the bilateral contract price is between them, and it is not necessary to disclose the price to the spot market or the operator. However, the energy 100 MWh will be delivered through the grid and the operator is informed of the energy quantity.

Outside the spot market, the buyer B pays the seller S \$50/MWh × 100 MWh = \$5,000. Then, both of them go to the spot market. Seller S tells the market operator that he or she will self-schedule a power injection of 100 MWh at node 1 during the hour. Buyer B tells the market operator that he or she will self-schedule a power withdrawal of 100 MWh at node 2. The quantity 100 MWh in the bilateral contract is considered as self-scheduled energy in the spot market. All self-scheduled energy for the hour, including bilateral contracts, electricity financial contracts, and so on, are informed to the operator before the spot market. The operator runs SCOPF market model for generation schedules, as well as obtaining LMPs for all nodes. The spot market clearing and LMP calculation examples have been introduced in Sections 8.3 and 8.4.

In this example, assume the LMPs obtained for node 1 and node 2 are \$35/MWh and \$45/MWh, respectively, or LMP_1 = \$35/MWh and LMP_2 = \$45/MWh. As the seller S has self-scheduled energy injection 100 MWh at node 1, and buyer B has self-scheduled energy withdrawal 100 MWh at node 2, same as other market participants, they should pay or be paid according to the nodal price. Then, in the market, S is paid at the price LMP_1 = \$35/MWh for the 100 MWh, which is \$3,500, and B pays at the price LMP_2 = \$45/MWh for the 100 MWh, which is \$4,500. The payment and the money flow is shown in Figure 8.8.

From the money flow shown in Figure 8.8, we can see that the buyer B has paid in total (\$5,000 + \$4,500) = \$9,500, and the seller S has been paid by (\$5,000 + \$3,500) = \$8,500. Buyer B pays twice, and seller S is paid twice. One payment is according to their bilateral contracts, and the other payment is according to the LMPs obtained in the spot market. So, there is a double payment for the owners of bilateral contracts or self-scheduled energy of financial contracts. The payment is shown in Table 8.13.

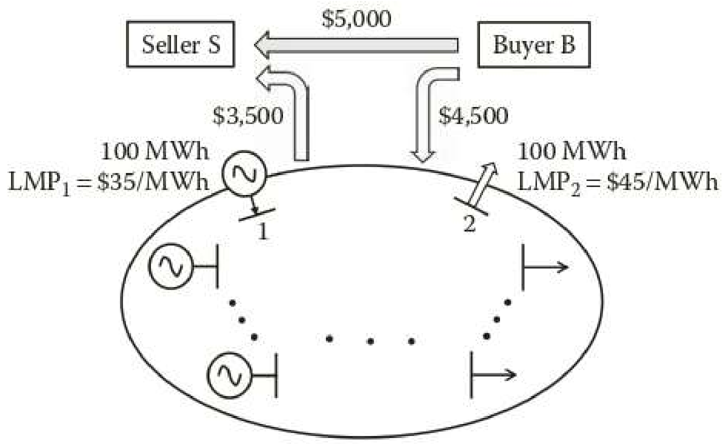

FIGURE 8.8 Double payment of bilateral contracts.

Table 8.13 Original payment (in \$) for seller S and buyer B

Payment (\$)	Seller S	Buyer B
Bilateral contract	+\$5,000	−\$5,000
Market settlement	+\$3,500	−\$4,500
Net settlement	+\$8,500	−\$9,500

There are two methods to offset the double payments in the market. One way is to offset the double payment internally through the market settlement system. The other way is through external agreements, such as CfD, to settle the double payment outside the market. The two methods will be introduced in Sections 8.5.3.2 and 8.5.3.3, respectively, followed by the examples.

8.5.3.2 Example II: Internal settlement for double payment offsetting The double payment can be offset internally in the market by selecting a trading hub. The trading hub can be an independent node, say node k. The selection of trading hub can be negotiated and determined by the seller S and buyer B. The nodes where the seller and buyer are located, node 1 and node 2 in this example, can also be selected as the trading hub for payment offset.

If a trading hub, node k, is selected. The LMP of node k obtained from the market clearing is \$39/MWh, as shown in Figure 8.9. Seller S and buyer B will use $LMP_k = \$39/MWh$ to offset the double payment. Seller S will pay back $\$39/MWh \times 100\ MWh = \$3,900$ to buyer B. Then, the net payment after offset is shown in Table 8.14.

The net settlement in Table 8.14 shows that the buyer pays \$5,600 and the seller gets paid \$4,600. The buyer pays \$600 more compared to the contract amount \$5,000 and the seller receives \$400 less. The differences are the payment for losses and congestions. The seller and buyer pay a total \$1,000 for losses and congestions. This amount is allocated to the seller and buyer by \$400 and \$600, respectively, using the LMP of hub k as the reference. In other words, the seller pays the loss and congestion cost between node 1 and node k, and

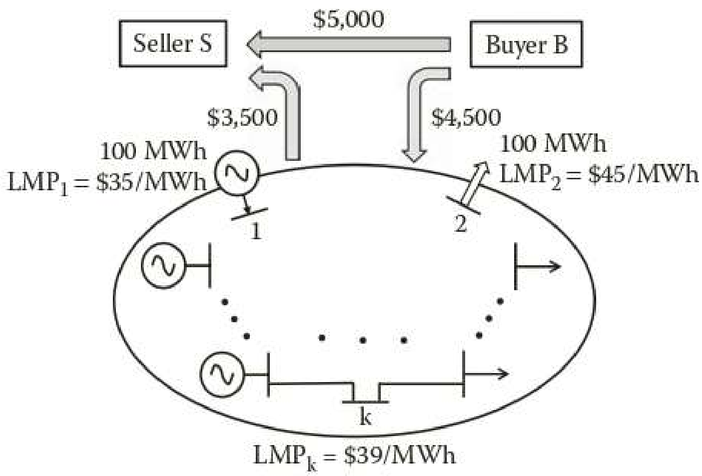

FIGURE 8.9 Offset the double payment using trade hub node k.

Table 8.14 Payment (in $) for seller S and buyer B using trade hub node *k*

Payment ($)	Seller S	Buyer B
Bilateral contract	+$5,000	−$5,000
Market settlement	+$3,500	−$4,500
Hub k settlement	−$3,900	+$3,900
Net settlement	+$4,600	−$5,600

the buyer pays the loss and congestion cost between node 2 and node k. If a different trading hub is selected, the amount for losses and congestions allocated to the seller and the buyer will be different. So, the selection of trading hub is negotiated and determined by the seller and buyer in the bilateral contract.

If node 2 is selected by the two parties as the trading hub, the net payment after offset is shown in Table 8.15.

The net settlement in Table 8.15 shows that the buyer B pays the amount $5,000 as in the bilateral contract, and the seller S receives only $4,000. By using node 2 as the trading hub, the loss and congestion costs are all paid by the seller S. Similar, if node 1 is selected as the trading hub, the buyer B will pay for all loss and congestion costs.

From earlier examples, it shows that market participants are facing the risk of high congestion cost, even if they have bilateral contracts to hedge the risk of high energy price. Transmission right is developed to hedge the risk caused by transmission congestions. In the example of using node k as the trading hub as shown in Table 8.14, if the buyer B holds transmission right from node k to node 2, it will be reimbursed the congestion cost, which is calculated according to the congestion components of LMP_2 and LMP_k. Similarly, the seller S can also purchase transmission right in advance to hedge the risk of high congestion costs due to transmission congestions. The concepts

Table 8.15 Payment (in $) for seller S and buyer B using trade hub node 2

Payment ($)	Seller S	Buyer B
Bilateral contract	+$5,000	−$5,000
Market settlement	+$3,500	−$4,500
Hub 2 settlement	−$4,500	+$4,500
Net settlement	+$4,000	−$5,000

of transmission right will be introduced in Chapter 9, about congestion management.

8.5.3.3 Example III: External settlement for double payment offsetting Besides the market internal settlement described in the previous section for double payment offsetting, external contract can be signed to offset the double payment while allocating the loss and congestion cost to the parties of bilateral contract. Financial tools, such as CfD, can be used to allocate the transmission payment owing to losses and congestions.

CfD payment occurs outside the market settlement system. It allows parties a flexibility in determining the settlement price to offset double payment. Parties may or may not link the offsetting transaction to a market price.

For example, in the CfD contract, the parties in Figure 8.8 can decide to use the LMP of node 1 or node 2 for offsetting transaction. If LMP of node 1 is decided for offsetting in CfD, seller S will need to pay $35/MWh $\times$ 100 MWh = $3,500 to buyer B outside the market after LMPs are published. The net payment for using LMP of node 1 in CfD is shown in Table 8.16. In this example, buyer B pays congestion and loss costs. In this case, buyer B knows in advance that he or she will pay for the congestion and losses in addition to the energy payment of bilateral contract. To hedge the risk, the buyer can purchase transmission right in advance. For example, if the difference in congestion components between LMP1 and LMP2 is $4, the buyer B will be reimbursed by $400 for congestion revenue. This hedges the risk caused by congestion but not loss component.

Other prices not related to LMPs in the market can as well be determined in the CfD contract. For example, if the price in the CfD for offsetting transaction is $xy (assume xy is a number that equals to $10x + y$), the net settlement for this CfD price is

Table 8.16 Payment (in $) for seller S and buyer B using LMP of node 1 in CfD

Payment ($)	Seller S	Buyer B
Bilateral contract	+$5,000	−$5,000
Market settlement	+$3,500	−$4,500
CfD using LMP_1	−$3,500	+$3,500
Net settlement	+$5,000	−$4,000

Table 8.17 Payment (in $) for seller S and buyer B using CfD for offsetting transactions

Payment ($)	Seller S	Buyer B
Bilateral contract	+$5,000	−$5,000
Market settlement	+$3,500	−$4,500
CfD price $xy	−x, y00	+x, y00
Net settlement	+$8,500−x, y00	−$9,500+x, y00

shown in Table 8.17. The two parties can determine the CfD price $xy outside the market.

8.5.4 Contract that is not resource specific

In the previous section, examples are given for settlement of bilateral contracts. In the examples, the seller owns generators and injects contracted energy to the grid. In an electricity market, it is possible that the parties of a bilateral contract do not actually inject energy into or withdraw energy from the grid. Contracts of this type are not resource specific, such as forward contracts and other electricity financial contracts. This type of contract provides possibilities for sellers with and without generation resources to speculate in the market.

In the examples of the previous section, the seller generates 100 MWh of contracted energy and injects into the grid. In the spot market, the seller is paid at the price $35/MWh, which is the LMP of node 1. It is possible that the production cost of the generator is $43/MWh. Although LMP is lower than its production cost, the seller hedged the risk by signing bilateral contract. From this viewpoint, the seller can purchase energy from spot market at a price $35/MWh instead of generating at a cost of $43/MWh to fulfill the energy contract. So, sellers with generation resources can maximize their benefits by bidding in the spot market and only run their units when prices are high enough to be profitable.

We use Example II (node k as trading hub) and the results in Table 8.14 and Figure 8.9 to illustrate the profit of seller S if the contract is not required with specific resources. The results are shown in Table 8.18. In rows 2–5 of the table, the net payment from the bilateral contract and market is given. The production cost of the seller is shown in row 6. The production cost is calculated as $43/MWh × 100 MWh = $4,300. The profit of the seller from the transaction is $300.

As the production cost is high, the seller S can bid in the spot market to purchase 100 MW from the market for the hour.

Table 8.18 Profit (in $) for contract without specific resource using trading hub node *k*

Payment ($)	Seller S	Buyer B
Bilateral contract	+$5,000	−$5,000
Market settlement	+$3,500	−$4,500
Hub *k* settlement	−$3,900	+$3,900
Net settlement	+$4,600	−$5,600
Production cost	−$4,300	
Profit	$300	

Table 8.19 Profit (in $) for contract without specific resource (no generation)

Payment ($)	Seller S	Buyer B
Bilateral contract	+$5,000	−$5,000
Market settlement	0	−$4,500
Hub *k* settlement	−$3,900	+$3,900
Net settlement	+$1,100	−$5,600
Production cost	0	
Profit	$1,100	

The self-scheduled 100 MWh injection and the purchase of 100 MWh at node 1 result in a zero injection at the node from the seller S. The seller does not need to generate electricity for the hour in this case. The profit of the seller is shown in Table 8.19. The result shows that the profit is $1,100, which is much higher than the results in Table 8.18, if the seller generates 100 MWh.

From the examples shown in Tables 8.18 and 8.19, we can see that even if the seller does not own generating units or the buyer does not actually consume energy, it is possible for them to have bilateral contracts or forward contracts. They can sell or purchase energy through long-term contracts and then buy back or sell in the spot market. Loss and congestion charges will be allocated to them according to the loss component and congestion component of LMPs.

8.6 Uniform zonal price-based market model

Uniform zonal pricing is the other commonly used pricing mechanism besides nodal pricing. Most ISOs in the United States adopt nodal pricing mechanism with a bid-based SCUC or SCOPF for spot market settlement. SCUC and SCOPF have been traditionally used in the United States for generation scheduling since centralized dispatch. It is straightforward to use SCUC and SCOPF and obtain nodal prices by calculating the cost sensitivity to each node. Loss components in LMPs usually are not big. If there is no congestion in the system, the values of nodal prices are close. Serious congestions could result in a big difference between LMPs. Nodal prices of a local area are usually similar unless a congestion occurs. For some systems, there are seldom congestions within a local zone, and nodal prices in the zone are similar most of time. It is possible to ignore losses and use one price for the zone to represent the prices of nodes in the zone. This is more or less similar to the concept of energy component of LMP. The pricing mechanism is called zonal pricing. Zonal price-based market model has been adopted by many systems, especially for systems with clear transmission paths between zones while the transmission is single in direction and forecastable most of the time. The system could be divided into zones based on frequently congested tie lines. One of the criterion to determine zones is that usually there is no congestion within the zone. Nordic electricity markets and European electricity markets adopted zonal price-based market model. For example, Nordic countries have plenty of hydro power in the North, and 99% of power generation in Norway is from hydro. Power is transmitted from areas with high percentage of hydro power (e.g., Norway and the North) to load centers in the South. Power transmissions are in single directions, and congestions occur only on some tie lines. Price zones are separated on the basis of frequently congested lines. For example, in current Nordic market, there are five price zones in Norway, four price zones in Sweden, one price zone in Finland, and two price zones in Denmark.

8.6.1 Uniform market price

The zonal price settled for a zone is obtained usually using uniform market price. Uniform market pricing is to set the offer price of the last selected seller as the market price. All selected sellers will be paid with the uniform price, although their offer prices are lower than the price. If the system demand is elastic, buyers in the market bid for energy consumption. The uniform market price will be the intersection point of the supply curve and the demand curve. The market price is set as the offer price of the last selected seller and the bid price of the last selected buyer, although offer prices are lower than the uniform price and bid prices are higher than the uniform price, as shown in Figure 8.10.

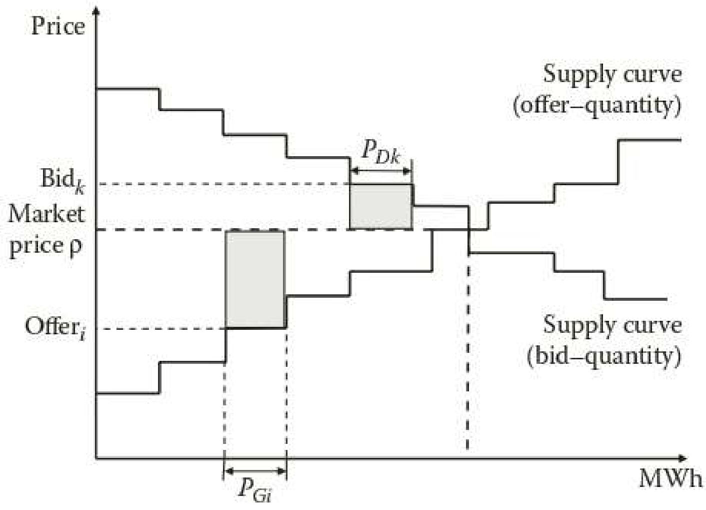

FIGURE 8.10 Market settlement with the uniform price.

The figure shows a simplified way to obtain uniform price. The sellers provide offer price–quantity pairs to the operator, and the buyers provide bid price–quantity pairs to the operator. Offer prices are ranked with an ascendant sequence and bid prices are ranked with a descendant sequence. Here, losses are ignored. It is possible to consider losses using correction factors when deciding the merit order. The intersection point determines the uniform market price. In the figure, the market price is ρ. For a seller i, its offer–quantity pair is the price offer_i and quantity P_{Gi}. In fact, the market price ρ paid to the seller is higher than its offer price offer_i. The seller receives an amount $(\rho - \text{offer}_i)\cdot P_{Gi}$ more than his expectation. The amount $(\rho - \text{offer}_i)\cdot P_{Gi}$ is shown as the shaded area in Figure 8.10. For a buyer k, its bid–quantity pair is price bid_k and quantity P_{Dk}. The buyer pays market price ρ, which is lower than his or her bid price bid_k. The buyer pays an amount $(\text{bid}_k - \rho)\cdot P_{Dk}$ less than his or her bid. The amount $(\text{bid}_k - \rho)\cdot P_{Dk}$ is shown as the shaded area in Figure 8.10.

The mechanism of uniform price was proposed and applied as early as in the power pool era. The purpose to pay with uniform price instead of pay-as-bid is to guarantee fairness in the auction process. Assume that a market clearing mechanism is designed as pay-as-bid, market participants would maximize their benefits in the auction, then sellers will try to offer as high as possible and buyers will try to bid as low as possible, as long as they can be selected by the market. Thus, the market is distorted, and the auction prices can hardly reflect the cost of generation. To encourage honest auction of actual cost, uniform pricing is designed. With this market clearing mechanism, sellers will be anyway paid with the highest selected offer price, so sellers have incentives to offer their prices as low as possible to be selected. However, a seller's offer will not be lower than the generation cost, thus to be sure that the profit will not be negative in case the offer is the last selected price and will be set as the market price. For similar reasons, buyers will bid as high as possible to show the actual values they can pay for purchasing the energy. After market clearing, they will pay at the uniform market price that is lower than their bids. In a market with uniform pricing, if the number of market participants is big enough (less market power), it is ok to assume that market participants would bid at their costs.

8.6.2 Zonal uniform market price

8.6.2.1 Potential price zones To apply zonal pricing, the first step is to have potential price zones. In the time period of light load, there is no congestion in the system, the whole system is a price zone with one uniform price. The price can be obtained using the method described in Section 8.6.1. If there are congestions during the time periods of heavy load, the system is separated into zones according to congested tie lines. At the stage of market design, the system operator and the regulator decide several potential price zones. The separation of price zones is based on frequently congested transmission corridors in the history. The lines that have congestion potentials connect two price zones. The reason we call them potential price zones is that the price zones are separated and visible only if there are congestions between zones. If there is no congestion, we do not see zones in the market. The zone separation is shown in Figure 8.11.

Electric energy is transmitted from the North to the South most of the time in the power system of Figure 8.11. For example, if generation cost in the North is cheaper than in the South, and there are more demands in the South (load center), buyers in the South would like to purchase energy from the North, which results in a large amount of energy being transmitted from the North to the South through the transmission corridors shown in the figure. For some scenarios, transmission lines of the transmission corridors could be overloaded by the transactions from the North to the South. The system is marked with three potential price zones, A, B, and C, as shown in Figure 8.11a. The criterion of separating the potential zones is that power flow in transmission lines between A and B and lines between B and C could reach

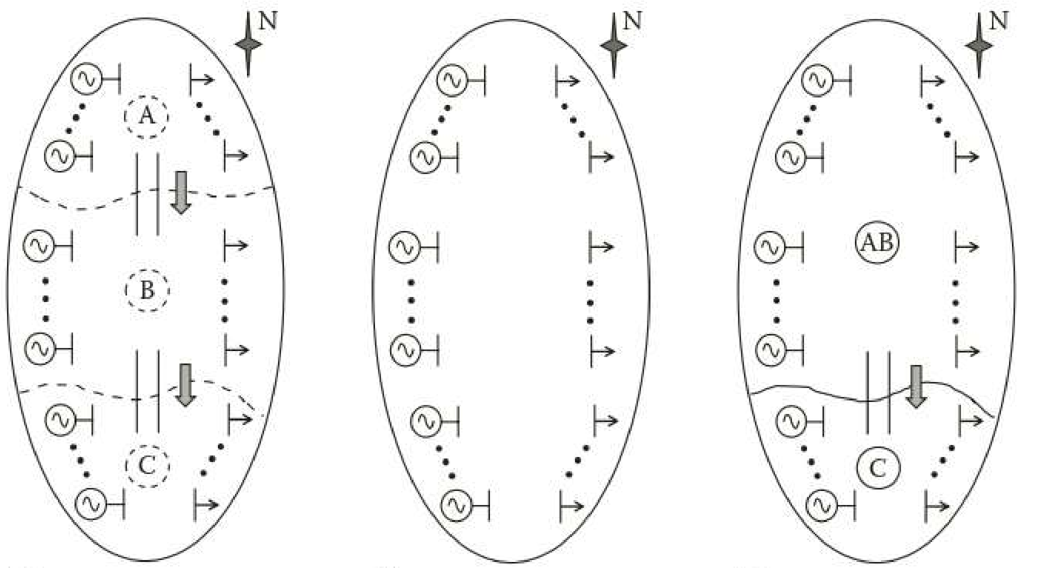

FIGURE 8.11 (a) Potential price zones, (b) no congestion, and (c) price zones separated for congestions.

their transmission capacities. The zones are potential zones (not visible unless congestion occurs).

8.6.2.2 Zonal price clearing If the transactions in the market during a time period do not result in any overloading (or congestions) between lines, the price zone of the system is set as one zone, as shown in Figure 8.11b. The whole system has one uniform price, which is obtained as the intersection point of the offer curve and the bid curve. The offer–quantity pairs from all sellers form the offer curve. The bid–quantity pairs from all buyers form the bid curve. Similar to the nodal pricing model, bilateral contracts are regarded as self-scheduled energy, as price-takers, in the uniform market price clearing.

If the transactions, including bilateral contracts, forward, futures, and sport market, result in transmission overloading of the tie lines between potential Zones B and C, the system is separated into two zones. The border is shown in Figure 8.11c. In this case, potential Zones A and B are combined into one price zone, Zone AB, potential Zone C forms the other price zone, Zone C. The power flow in the tie line across the border of two price zones is limited by the line capacity. Each price zone has its own zonal price. In Zone AB, all sellers' offers in Zone AB form the offer curve. All buyers' bids plus the power transmitted to Zone C through the tie line form the demand curve. The zonal price for Zone AB is obtained by the intersection point of the supply curve and the demand curve of Zone AB. In Zone C, all sellers' offers plus the power transmitted into Zone C through the tie line form the supply curve. All buyers' bids in Zone C form the bid curve. The zonal price for Zone C is obtained by the intersection point of the supply curve and the demand curve of Zone C. Similar to the nodal pricing market clearing, bilateral contracts here are regarded as self-scheduled energy and are price-takers in the uniform market price clearing. The power transmitted in the tie line, which is limited by the transmission capacity, is also price-takers in the clearing model.

Owing to the transmission limits between two price zones, certain amount of cheap electricity in Zone AB cannot be transmitted to Zone C (the load center), and expensive generators in Zone C need to be scheduled to supply electricity demand. So, the zonal price of Zone C is more expensive than the zonal price of Zone AB. Uniform price clearing for the system price (without congestion) and for the zonal prices (with congestions) is shown in Figures 8.12 and 8.13, respectively.

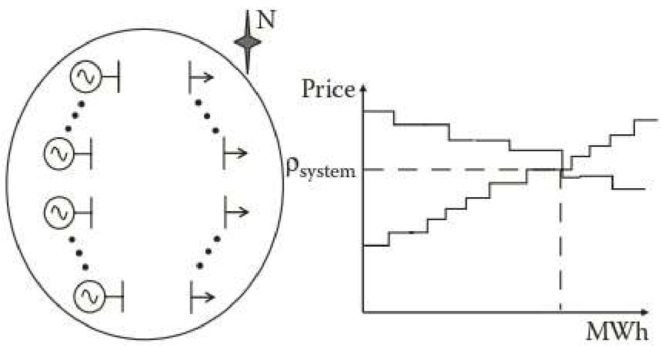

FIGURE 8.12 Uniform system price (without congestion).

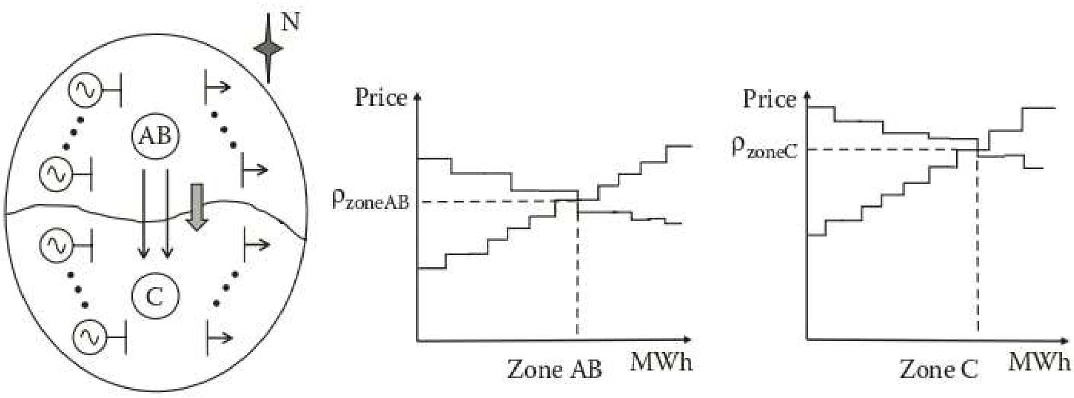

FIGURE 8.13 Zonal price (with congestions and price zone separation).

8.6.3 Zonal price market operation issues

Zonal pricing-based market design is a simple market model. The market price is determined by simplified merit orders of sellers and buyers. Prices are calculated according to zones. Market participants in one zone have the same price. Combination of price zones and the price differences between zones depend on security check and congestion management. In the market, long-term bilateral contracts and OTC contracts are still dominant. The market clearing includes the quantities of these long-term contracts. Zonal prices reflect congestion charges. Here, we introduce the operation procedure of a zonal pricing-based market.

Long-term contracts dominate energy transactions in most electricity markets. Long-term energy transactions are signed between two market participants, which are independent of the system operator. The contracts are signed as bilateral contracts, or signed in the OTC market. Once a long-term contract has been signed, the system operator is informed of the time and the energy quantity of the transaction, while the contract price is not necessarily disclosed. A preliminary security check will be implemented by the system operator to ensure the bilateral transaction satisfies operation requirements. In the spot market, sellers and buyers bid in the market with quantity–price pairs. The system operator accumulates all offer pairs and bid pairs and include quantities of long-term contracts in the supply and demand curve for clearing, as shown in Figure 8.14. The energy signed in the long-term contracts is considered as self-scheduled energy in the spot market clearing. They are price-takers in the spot market, which is settled as described in Section 8.6.2.2. The uniform system price settlement shown in Figure 8.12 is adjusted as shown in Figure 8.14 for including the amount of energy in long-term contracts. Once the market is cleared, the generations from sellers and the consumptions

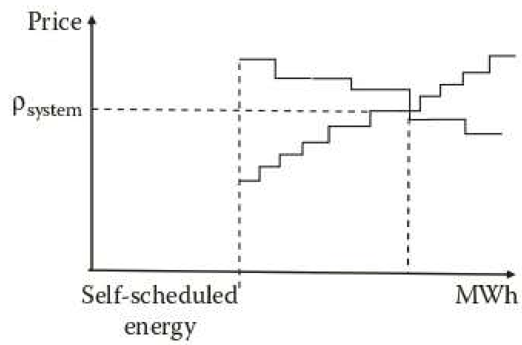

FIGURE 8.14 Uniform system price settlement with self-scheduled energy.

of buyers are obtained. The cleared price is the system price. Then, the system operator will do security check. All generations and consumptions selected in the spot market and signed in long-term contracts are input in the real network simulation model. The system operator runs power flow and checks if any line is overloaded. If no line is overloaded, the system is settled with the obtained system price. The whole system has one price for the hour. If tie lines between potential price zones are overloaded and congested, the system will be divided into zones accordingly. Zonal prices are obtained as suitable for Section 8.6.2.2 and Figure 8.13. Similar to in Figure 8.14, the adjustment due to self-scheduled energy as applied to zonal price settlements is also described in Figure 8.13.

In the market, congestions result in different zonal prices for neighboring two zones linked by the congested line. Congestion charge is calculated as the product of zonal price difference and transmitted power. If the seller and the buyer of a bilateral contract are located in different price zones, they need to share the congestion charge caused by the transaction. The congestion charge is shared by two parties of the transaction. A common way to allocate the congestion charge is to sign a CfD to determine the allocation of congestion cost as a supplementary agreement when the bilateral energy contract is signed.

The energy traded in each hour is composed of long-term transactions and spot market transactions. We use Figure 8.15 to illustrate an example of energy transactions for a 24-hour time period.

Figure 8.15 shows that long-term bilateral contracts and other self-scheduled energy form the fundamental electric

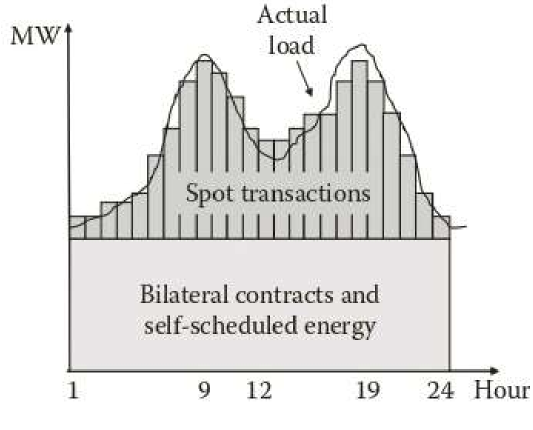

FIGURE 8.15 An example of energy transactions for 24-hour time periods.

energy supply. Hourly energy transactions are traded in the spot market and determine the hourly spot market price. However, the hourly-ahead schedule deviates from real-time demand. The actual load curve is shown in this figure. The differences between the hourly-ahead energy schedule and the real-time actual load are compensated in the balancing market run by the transmission system operator (TSO), which will be introduced in Chapter 10 about ancillary services.

In a zonal price-based market, energy is traded in the spot market, OTC market, and bilateral transactions. Transmission companies provide transmission services to market participants. Transmission companies do not get paid directly from the market except congestion charges if congestion occurs. In the market design, market participants pay transmission fees to the transmission company according to their locations, grid-connection points, voltage levels, and so on. The design of transmission tariff will be introduced in Chapter 9.

Uniform zonal price-based electricity market has a simple market design. Frequency regulation, balancing, and reserves are considered in a separate market from energy market. Transmission tariffs are charged separately. The market clearing is simple by applying uniform pricing mechanisms. Congestion is managed by separating price zones. In the market design, defining potential price zones (or congestion tie lines) is critical. Well-designed price zones are the basis of successful market operations. The zones are usually determined according to the topology of the network, typical load flow, and experiences from practical system operations.

8.7 Nodal pricing versus zonal pricing

In this chapter, we have introduced nodal pricing-based market and zonal pricing-based market. They are the major types of market designs in the world applied in different power systems. There is no standard answer about which type of market is better. When system operators decide to adopt nodal pricing or zonal pricing, they should select and apply the one that fits their systems. There are several considerations when making decisions. System operators need to consider if the network is a meshed network or a system with long distance transmission corridors, whether the system has frequent congestions, as well as the power flow directions of congested lines. Besides, it also depends on the existing dispatch method, for example, centralized dispatch or decentralized dispatch, and whether the dispatch is security-constrained dispatch already. Market power in the network is also a factor for consideration.

There is no standard answer of which market design is most suitable to a power system without testing. Both nodal and zonal pricing could succeed or fail in a system. The power industry has learned from practical lessons. For example, PJM adopted zonal pricing-based market model before 1997, but it experienced a collapse of zonal pricing when some generators self-schedule themselves to constrained/congest transmission lines. In 1998, PJM changed to nodal pricing-based market and eliminated the market power of gaming the security constraints. The ISOs in the United States, PJM, ISO New England, New York ISO, and the market of New Zealand use nodal price-based market. It is straightforward that they use transmission right for congestion management and transmission charges. The concepts of transmission rights will be introduced in Chapter 9.

In Australia and Nordic countries, price difference is small within congestion zones, and congestions occur mainly on the interconnection tie lines. Australia market and Nord Pool use zonal pricing for market design and congestion management. Their systems have some common characteristics. For example, there is less congestion within a zone, and the nodal prices within a zone are very close; congestions occur only on certain tie lines; power flow directions on the tie lines are fixed most of the time.

8.8 Summary

Electricity market models and electricity pricing mechanisms are introduced in this chapter. Two common pricing mechanisms, nodal pricing and zonal pricing, and their market operations are illustrated in the chapter. Mathematical models of nodal pricing-based electricity market as well as the procurement of locational marginal prices are presented and derived. Central dispatch-based SCOPF market model is formulated. Locational marginal price formulations are derived from the market model. Examples are provided to explain market settlement procedures, as well as how to obtain market prices for both nodal price-based market and zonal price-based market. This chapter provides a clear description of electricity market models and their mathematical formulations, which are the basis for further research in electricity market.

CHAPTER NINE

Congestion management and transmission tariff

9.1 Introduction

Electricity is transmitted through transmission network. Energy transactions are implemented only if the energy can be delivered from generators to customers. Transmission availability is the necessary condition for energy trading. Market operation is subject to transmission availability. If electricity can be distributed freely in a network, the marginal value of electricity at any location in the network equals to the system marginal price, which is obtained as the intersection point of offer curve and bid curve. However, power flow in the network is restricted by physical laws and network parameters. Power grid topology and line impedances determine power flow in each line. Due to security operation requirement, power flow in each line should be controlled under the line transmission capacity limit. When market participants sign long-term contracts and submit price–quantity pairs in the spot market, they do not consider available transmission capacities or system security. The system operator does system security check in the day-ahead spot market, and guarantee market transactions fulfill transmission limits and other security limits. For example, in nodal price-based market, security-constrained optimal power flow or security-constrained unit commitment (UC) is run for day-ahead spot market clearing and generation scheduling. In zonal price-based market, security check is implemented in market clearing to ensure no transmission line is overloaded. To manage transmission constraint violations, price zones are separated and zonal prices are cleared within price zones. In both types of markets, it shows that price differences reflect transmission congestions and constrained line limits. The price at the sending end of a constrained line is lower than the price at the receiving end. Congestion management is an important part of electricity market operation for satisfying system operation requirements. In this chapter, we will first introduce congestion management in both nodal price market and zonal price market. Then, we will introduce transmission charges in different types of markets for economic compensations to transmission services provided by transmission companies.

9.2 Congestion management

9.2.1 Transmission congestion

Transmission congestion and congestion management are new terms after power systems moved to market operation. In traditional power system operation, system security and reliability are the major considerations in generation scheduling. Security analysis and stability analysis are implemented in the energy management system. In electricity market operations, the market operator runs the market according to long-term electricity trading agreements and auctions in the spot market. Market participants are independent and not responsible for system security operation. Their auctions and transaction agreements submitted to the market operator do not have any security considerations. Simply aggregating the auctions and transactions may result in operation issues, for example, overloading of power lines, and so on. It is necessary to implement security check and manually intervene the market clearing to ensure that the market operation and generation schedule will not lead to any transmission line overloading or transmission congestions. In electricity market, it is called *congestion management*. Congestion management is a way to adjust market schedules to satisfy power system operation requirements. The system operation constraints enforced on the market operation usually result in additional costs compared to a market without system operation constraints. The additional cost is *congestion cost*.

9.2.2 Congestion management and congestion charge

In most electricity market designs, transmission constraints and security operation requirements have been considered in the market clearing. The profile of spot market prices for 1 hour reflects transmission congestions in the market. If the market transactions do not cause congestions, in the zonal pricing-based market, the whole market has a uniform system price and there are no separated price zones; in the nodal pricing-based market, nodal prices would be the same and equal to the system marginal price, if losses are ignored. Given an economic definition, in the market, the path from A to B is congested if electricity price at B is greater than the electricity price at A. The economic definition applies to both zonal pricing and nodal pricing markets.

Congestion cost is evaluated according to the differences of two nodes or two zones. The bilateral contract holders located at two different nodes/zones share the congestion cost allocated to the transaction, which is the price differences of the two nodes/zones. The presence of transmission line capacity constraints leads to higher marginal cost. Market participants pay more if congestions occur in the network. Congestion management is necessary for the market operation and the allocation of congestion costs.

There are two methods for congestion management: locational/zonal pricing and transmission right. Locational pricing/zonal pricing for spot energy is the market approach to short-term congestion management. Transmission right is the long-term financial method for congestion management.

Locational (or nodal) pricing and zonal pricing have been introduced in Chapter 8. The transactions leading to transmission congestions and greater price differences between source and sink nodes are responsible for higher congestion costs. Congestion charges to market participants provide them economic incentives to adjust their transactions that lead to congestions. Locational price and its congestion component are market signals of transmission congestions. In a short-run, locational pricing and market settlement provide market approaches for congestion management. This has been introduced in Chapter 8 with the market models.

9.3 Transmission right

Transmission rights are financial methods for long-term congestion management for a nodal pricing-based market. Transmission rights are traded in a separated market from energy market. Fundamentally, transmission right is a financial tool to hedge the risk of transmission congestion. Sellers and buyers need to own transmission rights if they plan to transmit energy from one node to the other node. The economic basis of transmission right is the price difference between the locations of seller (node i) and buyer (node j). For example, if the price difference between two nodes is estimated to be $\Delta\lambda_{ij}$ per

MWh, the buyer at j may like to pay the amount $\Delta\lambda_{ij}$ to buy the transmission right for buying power from i. Similarly, the seller may also like to pay a certain amount to buy the transmission right for selling power to node j. Moreover, transmission rights are financial tools; the owner of a transmission right can use it or make profit from it.

There are different types of transmission rights designed in financial markets. Here, we introduce several common transmission rights: physical transmission right, FTR, power flow-based flow-gate right, and TCC.

Physical transmission right is a practical method for congestion management. It is a supplement to locational pricing in managing transmission congestions. The holders of physical transmission rights have the right to inject a certain amount of energy at node i and withdraw the energy at node j. Physical transmission rights allocate the rights for holders of utilizing the transmission capacities between nodes. Once physical transmission rights are purchased, the owners are guaranteed to be scheduled to utilize the right for power transmission on the line between the nodes. The transmission rights are used and exercised physically by the holders in the market operation. Another similar transmission right is transmission congestion contract (TCC), which defines and allocates transmission rights from point to point.

Different from physical transmission right, financial transmission right (FTR) is a financial instrument. The holders of FTRs are guaranteed the financial equivalent of using the transmission right, but not necessarily the use of the capacities of physical lines. Moreover, the value of FTR is independent of the actual power flow on the line. It depends on the transmission congestion in the network and the severity of congestion. FTRs are traded in a separate market from the energy market. The purpose of purchasing FTR is to use financial instrument to hedge the risk of high locational marginal prices (LMPs) raised by transmission congestions between nodes. Buyers and sellers in the energy market can buy FTRs in advance to hedge risks of prices and congestions. As a financial tool, FTRs are not compulsory for the owners to use; they can trade FTRs and make profits from them.

Flow-gate right (FGR) is a power flow-based transmission right. It is commonly used in locational pricing-based electricity markets. The design of FGR is to the maximum extend to match the actual power flow. However, given a complicated power network, it is not easy to allocate transmission rights to lines, just like it is not straightforward to allocate system losses to power transactions. Sensitivity analysis and power transmission distribution factor (PTDF) are traditionally used for the loss allocation. Similar methods are applied in flow-gate-based transmission rights. PTDF of DC load flow can be used to define the flow-gate transmission right to match the power flow. Furthermore, key flow gates are introduced in advance to forecast the path that congestion may occur. FGR is power flow based, and its design is more close to that of the physical power flow. It is straightforward to apply FGRs in electricity market.

Here, we make simple comparisons between FTR and FGR.

- FTR is defined as point-to-point transmission right; FGR is path-based transmission right.
- FTR is centralized dispatch-based transmission right; FGR is distributed and it is decentralized dispatch-based transmission right.
- Being a point-to-point transmission right, FTR could be negative, while FGR can only be positive since it is path based.
- FTR is given indirectly and settled after energy transactions and LMPs are cleared; FGR is given directly and settled in advance.
- Design of FGR is more complicated than that of FTR, while FGR has better market fluidity than FTR.

9.4 Transmission tariff

In deregulated power systems, transmission networks are owned by transmission companies. Generation companies are independent of transmission network. Transmission companies provide transmission services to the market for energy delivery. Transmission companies should be compensated for providing the services. In energy markets, sellers and buyers are settled for their energy transactions. Transmission costs are not included in the bilateral contracts or market auctions. Transmission companies recover their costs for energy delivery outside energy markets. There are several ways for charging transmission fees, for example, transmission tariff, congestion revenues, transmission rights, and so on.

To open the electricity market to all participants, open access of transmission and distribution network is a must. All market participants should be able to connect to the grid and trade energy through the grid without any entry barriers. Transmission and distribution tariffs are designed in parallel to electricity market design to ensure market participants are able to choose their counterparties freely in the network. Design of transmission tariff differs in different systems. Two common transmission tariffs are point-to-connection tariff and channel tariff. Channel tariff is a straightforward method for transmission charging. Transmission fee is charged on the basis of the actual transmission cost between two points. Similar to flow-gate-based transmission right, it is complicated to allocate transmission cost to a path. Compared to channel-based transmission tariff, point-to-connection tariff is simpler and makes it easier for customers to access the market. Point-to-connection tariff is a uniform payment for utilization of transmission grid regardless of transmission path and distance. Market participants at the same voltage level within an area pay the same transmission tariff. Point-to-connection tariff is beneficial to market open access. Market participants are able to know their transmission costs before signing energy transactions or bidding in the market. While the transmission cost for a market participant does not depend on the location of the counter party of the transaction. It is simpler for market participants to utilize the network for energy trading.

For most point-to-connection tariffs currently applied in electricity markets, different rates are applied for injecting power into the grid and withdrawing power from the grid. The rates may also depend on the geographic locations, which relate to the energy adequacy of the area. For example, if power is transported from the source area with sufficient power generations to the sink area, which is load center, the rate for a generator to connect from the source area is higher than that to connect from the sink area, which encourages new generations connecting to load centers. On the contrary, the rate for a load connecting to the load center is higher than a load connecting in the source area, which discourages additional electricity demand in the heavy load area. In some markets, sellers and buyers are charged for different point connection tariffs. Transmission and distribution (T&D) tariff areas are formed according to demand/buyer side, and generation/seller side, individually. The T&D tariff areas designed for buyers and sellers could look totally different in a map. A market participant is charged T&D tariff according to which tariff area it is located in the buyer's tariff area map if it purchases energy from the grid. If the market participant sells energy to the grid, it is charged T&D tariff according to which tariff area it is located in the seller's tariff area map.

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